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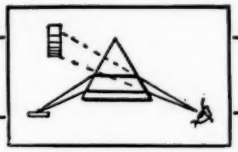

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On the Calculation of Absorption Coefficients of Daylight in Natural Waters

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(Received January 3, 1938)

The calculation of the true absorption coefficient of daylight in natural waters is discussed. It is shown that the vertical absorption coefficient K defined by $K=-1/I(dI/dz)$, where I is the intensity of light and z the vertical depth, is related to the true absorption coefficient k, defined by $k=-1/I(dI/dr)$, where r is the path length in the direction of the light, by the expression:

$$K=k\frac{\int_0^{2\pi}\int_1^a J'(\vartheta,\varphi)\frac{e^{-kxz}}{x}\,dxd\varphi}{\int_0^{2\pi}\int_1^a J'(\vartheta,\varphi)\frac{e^{-kxz}}{x^2}\,dxd\varphi},$$

where $J'(\vartheta,\varphi)$ is the intensity per unit solid angle per unit area just below the surface, ϑ and φ are the angles of refraction and azimuth respectively, x is defined by $x=\sec\vartheta$, and a is a number which depends upon the total amount of light reflected at the surface and the index of refraction. The integrals are evaluated for the special case that $J'(\vartheta,\varphi)$ is a constant, corresponding to the conditions prevailing on a perfectly cloudy day, and it is found that near the surface K must be increased around 20 percent of its measured value to give k.

THE penetration of light in natural waters, particularly the sea, has received considerable attention in recent years. Measurements are usually reported in terms of percent submarine intensity relative to surface intensity. From these data vertical absorption coefficients[1] may be calculated. It is the purpose of this paper to discuss certain considerations involved in the calculation of these coefficients when one takes into account the nature of the light incident on the surface. In practice most measurements are made with the light sensitive element in a horizontal plane. In general light may be incident on this plane from all directions and we ask the question: given a set of determinations of the total light flux across such a plane at various depths, what conclusions may be drawn concerning the absorption of light from a given direction?

The passage of diffuse radiation through an absorbing medium has been treated by several authors. Millikan and Cameron[2] have considered the intensity relations at different altitudes for cosmic rays incident from all directions on a spherical ionization chamber. Using tables given by Gold[3] they deduced values of the true absorption coefficients. Gross[4] gives a relation between

[1] Called by some workers vertical transmissive exponents and by others vertical extinction coefficients.

[2] R. A. Millikan and G. H. Cameron, Phys. Rev. **28**, 851 (1926).
[3] E. Gold, Proc. Roy. Soc. **82**, 43 (1909).
[4] B. Gross, Zeits. f. Physik **83**, 214 (1933).

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the measured intensity of cosmic rays traversing a spherical chamber and the intensity of the vertical rays. Maris[5] has considered the absorption of diffuse radiation in the atmosphere as observed by a plane detector. Due to the fact that the problem of the penetration of diffuse radiation in water is complicated by the presence of a refracting surface the results deduced in this paper differ in considerable detail from those referred to above.

Consider a horizontal surface at a depth z in water of index of refraction n. The light crossing this surface consists of contributions from (*a*) direct sunlight and (*b*) light scattered from clouds and the sky.

For a given wave-length the intensity, I_λ, of direct sunlight is given by the expression:

$$I_\lambda = I_{0\lambda} e^{-k_\lambda r}, \tag{1}$$

where $I_{0\lambda}$ is the intensity at the surface and k_λ is the absorption coefficient defined by $k_\lambda = -1/I_\lambda(dI_\lambda/dr)$, dr being an element of path length in the direction of the light. We define the vertical absorption coefficient, K_λ, by $K_\lambda = -1/I_\lambda(dI_\lambda/dz)$ where dz is a vertical element of depth. If ϑ is the angle the light in water makes with the vertical, the vertical absorption coefficient will be related to k by:

$$K_\lambda = k_\lambda \sec \vartheta. \tag{2}$$

k is not a pure absorption coefficient.[6] If we take account of scattering $k = \mu + \alpha$, where μ is the true absorption coefficient (the absorbed energy going into heat) and α is the scattering coefficient. Light may reach a given element of water either directly or be scattered to it from water molecules and particles suspended in the water.

The contribution of light scattered from the sky and clouds may, in some cases, comprise the major fraction of the light incident on the surface. Hence it is of interest to investigate the effects of such light in some detail.

We consider first the ideal case of a smooth surface and homogeneous water. We also treat only the light which proceeds directly from the surface to the element under consideration. The effect of scattering will be mentioned later. Let $J(\chi, \varphi)$ be the intensity per unit solid angle per unit area normal to the ray for a particular direction defined by χ, the angle of incidence, and φ, the azimuthal angle. Let us fix our attention on an element of area, dA, of the surface, which is defined by the angles ϑ and φ with respect to the point P (Fig. 1). The total flux passing through this surface in all directions per unit area is:

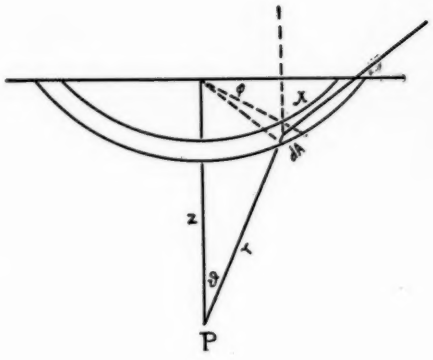

FIG. 1.

$$\int_0^{2\pi} J(\chi, \varphi) \cos \chi d\Omega.$$

We can consider this area to be itself an emitter of light to regions below, with an intensity of emission per unit solid angle per unit area of $J'(\vartheta, \varphi)$, where ϑ is the angle of refraction. Since the refracted ray is in the plane of incidence, φ remains unchanged. Assume provisionally that:

$$J'(\vartheta, \varphi) \cos \vartheta d\Omega' = J(\chi, \varphi) \cos \chi d\Omega, \tag{3}$$

which implies that the light flux across the surface is completely conserved. We shall show below how an approximate treatment of reflection losses can be made.

By Snell's law we have $\sin \vartheta = (\sin \chi)/n$ and hence $\cos \vartheta d\vartheta = (\cos \chi d\chi)/n$, where n is the index of refraction. Noting that $d\Omega = \sin \chi d\chi d\varphi$ and $d\Omega' = \sin \vartheta d\vartheta d\varphi$ we obtain:

$$J'(\vartheta, \varphi) = n^2 J(\chi, \varphi). \tag{4}$$

In water at the point P a distance r from the emitting area the light flux per unit horizontal

[5] H. B. Maris, Terr. Mag. **33**, 245 (1928).

[6] We shall discard the subscript λ in the following discussion. The reader shall understand that we refer always to a particular spectral band unless otherwise stated.

area due to the element dA of the surface is:

$$dI = J'(\vartheta, \varphi)(1/r^2)e^{-kr}\cos\vartheta dA, \qquad (5)$$

where $dA = z^2 \tan\vartheta \sec^2\vartheta d\vartheta d\varphi$. Making the substitution $r = z\sec\vartheta$ and integrating, we have for the total flux:

$$I = \int_0^{2\pi}\int_0^{\vartheta} J'(\vartheta, \varphi)e^{-kz\sec\vartheta}\cos\vartheta\tan\vartheta d\vartheta d\varphi. \qquad (6)$$

For an unshielded surface the upper limit ϑ will be ϑ_c, where ϑ_c is the critical angle given by: $\vartheta_c = \sin^{-1} 1/n$. In case the defining angle of the instrument is less than ϑ_c we must use this defining angle as the upper limit. If we make the substitution $x = \sec\vartheta$, and noting that x_c corresponding to ϑ_c is given by $x_c = \sec\{\sin^{-1} 1/n\} = n/(n^2-1)^{\frac{1}{2}}$ we have the expression for the intensity:

$$I = \int_0^{2\pi}\int_1^{n/(n^2-1)^{\frac{1}{2}}} J'(\vartheta, \varphi)\frac{e^{-kzx}}{x^2}dxd\varphi. \qquad (7)$$

If we have to deal with several spectral bands expression (7) must be integrated over the spectrum involved.

Using the definition of the vertical absorption coefficient: $K = -1/I(dI/dz)$ and substituting the integral expression for I we have the relation between K and k:

$$K = k\frac{\int_0^{2\pi}\int_1^{n/(n^2-1)^{\frac{1}{2}}} J'(\vartheta, \varphi)\frac{e^{-kzx}}{x}dxd\varphi}{\int_0^{2\pi}\int_1^{n/(n^2-1)^{\frac{1}{2}}} J'(\vartheta, \varphi)\frac{e^{-kzx}}{x^2}dxd\varphi}. \qquad (8)$$

It is to be pointed out that the expressions for I and K may also include direct sunlight by properly defining $J'(\vartheta, \varphi)$. In the special case that skylight is omitted from consideration, $J'(\vartheta, \varphi)$ is zero except for values of x between x and $x+\Delta x$, and for φ between φ and $\varphi+\Delta\varphi$, in which range it is given the constant value J_s'. Then

$$I = J_s'\frac{e^{-kzx}}{x^2}\Delta x\Delta\varphi \quad \text{and} \quad -\frac{dI}{dz} = kJ_s'\frac{e^{-kzx}}{x}\Delta x\Delta\varphi.$$

We obtain for K, $K = kx = k\sec\vartheta$ which is the expression previously stated for the case of direct sunlight.

In general $J(\chi, \varphi)$ and hence $J'(\vartheta, \varphi)$ will be of complicated form and values of K can only be obtained by graphical integration. One special case is of particular interest, namely: the case for which $J'(\vartheta, \varphi) = J$, a constant independent of ϑ and φ. This is the condition prevailing under a "perfectly" cloudy sky. We have:

$$K = k\frac{\int_1^{n/(n^2-1)^{\frac{1}{2}}}\frac{e^{-kzx}}{x}dx}{\int_1^{n/(n^2-1)^{\frac{1}{2}}}\frac{e^{-kzx}}{x^2}dx} \qquad (9)$$

$$\text{or} \quad \frac{K}{k} = \frac{\int_1^{n/(n^2-1)^{\frac{1}{2}}}\frac{e^{-kzx}}{x}dx}{\int_1^{n/(n^2-1)^{\frac{1}{2}}}\frac{e^{-kzx}}{x^2}dx} = f(kz, n).$$

As noted above, reflection losses at the surface have been neglected. For a smooth surface the fraction of light, F, transmitted is given by Fresnel's equation:

$$F = 1 - \left\{\tfrac{1}{2}\frac{\sin^2(\chi-\vartheta)}{\sin^2(\chi+\vartheta)} + \tfrac{1}{2}\frac{\tan^2(\chi-\vartheta)}{\tan^2(\chi+\vartheta)}\right\}, \qquad (10)$$

where χ and ϑ are the angles of incidence and refraction, respectively. A strict analysis would require that in the relation between $J'(\vartheta, \varphi)$ and $J(\chi, \varphi)$, $J(\chi, \varphi)$ be multiplied by this transmission factor. This would seriously complicate the mathematics of the situation and the following approximate treatment is outlined.

The total flux per unit area at the surface from a given direction interval is:

$$dI = J(\chi, \varphi)\sin\chi d\chi d\varphi \qquad (11)$$

and the amount transmitted is:

$$dI' = FdI. \qquad (12)$$

For $J(\chi, \varphi)$ constant we can integrate with respect to φ over the interval and obtain:

$$dI' = FI\sin\chi d\chi, \qquad (13)$$

where $I = 2\pi J(\chi)$.

Choosing units for $J(\chi)$ such that $I = 1$, the expression $F\sin\chi$ has been plotted for incident angles up to 90°. The area under this curve is

the total amount of light transmitted, I. The total light reflected is $I-I'$. We can determine an angle, χ_a, such that the area under the curve for $F \sin \chi$ from χ_a to 90° is numerically equal to $I-I'$. For $n=1.33$, χ_a 79°. If in the expression for the intensity, and hence for K, we take as an upper limit not the x determined by the critical angle but an x given by

$$x=\sec\{\sin^{-1} 79°/n\}=a$$
$$=1.48$$

we shall have corrected for reflection losses. We have then:

$$\frac{K}{k}=\frac{\int_1^a \frac{e^{-kzx}}{x}dx}{\int_1^a \frac{e^{-kzx}}{x^2}dx}. \tag{14}$$

The above integrals are not subject to direct integration, but can be evaluated from tables. Make the substitution: $kzx=u$ and $kz=y$. Then:

$$\frac{K}{k}=\frac{\int_y^{ay} \frac{e^{-u}}{u}du}{y\int_y^{ay} \frac{e^{-u}}{u^2}du}. \tag{15}$$

Values of the integral

$$\int_y^\infty \frac{e^{-u}}{u}du$$

are listed in the *British Association Tables*, Vol. I (1931), for values of y from 0 to 15. Noting that

$$\int_y^\infty \frac{e^{-u}}{u^2}du=\frac{e^{-x}}{y}-\int_y^\infty \frac{e^{-u}}{u}du. \tag{16}$$

values of the denominator in (15) can be easily calculated. Dr. G. F. McEwen of the Scripps Institution of Oceanography has worked up tables giving the values of both integrals which he very kindly put at my disposal. In Table I are given values of K/k for various values of a and y sufficient to cover a wide range of conditions. It can be easily shown that

$$\lim_{y\to 0} K/k=\log_e a/(1-1/a)$$

and that

$$\lim_{y\to\infty} K/k=1.$$

The expression for K cannot be solved explicitly for k and one must use a method of successive approximation. Suppose a measured value of K is 0.150 per meter at 10 meters depth, and suppose a has been determined to be 1.5. To choose y first set $y=Kz=1.5$. Interpolating in the table we find K/k to be 1.183. Choose $k=0.150/1.183=0.127\cdot y=kz$ is hence 1.27. For $y=1.27$, $K/k=1.19$, and $k=0.126$.

If the water is not homogeneous the above analysis must be applied successively. If k is some function of the depth, $k(z)$, then for a particular depth, $z=S$, Eq. (9) becomes:

$$K(s)=k(s)\frac{\int_1^{n/(n^2-1)^{\frac{1}{2}}} \frac{e^{-x\int_0^S k(z)dz}}{x}dx}{\int_1^{n/(n^2-1)^{\frac{1}{2}}} \frac{e^{-x\int_0^S k(z)dz}}{x^2}dx}. \tag{17}$$

To determine $k(S)$, given the measured value of $K(s)$, one must evaluate $k(z)$ by applying the equation to successive depths.

It has been pointed out that the contribution of light scattered beneath the surface has been neglected. If the scattered light were not itself absorbed, then any receiving surface would receive as much scattered light as had been scattered from the rays proceeding directly to it from the surface. In this case the calculated values of k would be those for pure absorption alone. If the pure absorption coefficient is large compared with the scattering coefficient, the contribution of scattered light would be small and our calculated k would be that observed for a thin beam of parallel light, where the scattered light is completely lost from the beam.

TABLE I. *Values of* K/k.

y \ a	1.1	1.2	1.3	1.4	1.5	1.6	1.7	1.8	1.9	2.0
0	1.048	1.094	1.137	1.178	1.216	1.253	1.289	1.323	1.355	1.386
.01	1.048	1.094	1.137	1.177	1.216	1.253	1.288	1.322	1.354	1.386
.02	1.048	1.094	1.137	1.177	1.216	1.253	1.288	1.321	1.354	1.385
.10	1.048	1.093	1.136	1.176	1.214	1.250	1.285	1.318	1.331	1.379
.50	1.048	1.092	1.133	1.171	1.208	1.239	1.269	1.298	1.324	1.349
1.0	1.047	1.090	1.129	1.165	1.196	1.225	1.251	1.274	1.295	1.314
2.0	1.045	1.087	1.121	1.137	1.170	1.199	1.218	1.232	1.245	1.255
5.0	1.044	1.080	1.102	1.119	1.131	1.139	1.144	1.147	1.149	1.150

The amount of scattered light actually present in sea water has been investigated by several authors. Hulburt[7] states that for pure sea water the scattering coefficient is less than one-third the true absorption coefficient. Pettersson,[8] by use of a scattering and transparency meter, finds bands of water marked simultaneously by strong "extinction" and by strong scattering coefficients. Powell and Clarke[9] find, for example, that in Provincetown Harbor, Mass., the upward component of light was 4 percent of the downward component at 0.5 m, increasing to 6 percent at 14 m. They also found that the ratio of maximum horizontal illumination to vertical illumination was 16 percent at 1.5 m and 17 percent at 10 m, while in the Gulf of Maine this ratio was 6 percent at 2 m and 20 percent at 15 m.

Two limiting cases for the incident light are: (a) all the light is concentrated in direct sunlight and (b) the light is completely diffuse. One effect of waves on the surface would be to make the light penetrating the surface more diffuse, as will scattering below the surface. The true value of k will hence lie between the k's obtained under these limiting conditions, and in practice will tend toward the values given by the analysis for diffuse illumination. Atkins and Poole[10] find that under certain conditions the angle of incident light is unimportant, which would signify that on passage through the surface the light becomes diffuse. A second effect of the waves will be to allow light to penetrate at angles greater than the critical angle (as reckoned for a smooth surface). The diffusion effect will not affect the results obtained for light which is initially diffuse. The effect of light penetrating at angles greater than the critical angle would have to be accounted for by increasing the limits of integration in Eq. (7). More exact knowledge of the distribution of light beneath the surface under varying conditions of sea and sky is required to determine the magnitude of this effect on the computed values of k. Since, however, the result of considering angles greater than the critical angle will be to increase the limits of integration, the values taken in the above analysis will be lower limits to the actual case of a rough surface.

The above analysis has been applied to data obtained in the Pacific Ocean off the coast of Southern California. An account of this work is to appear in a Bulletin of the Scripps Institution of Oceanography. The author is indebted to Mr. R. F. Gordon for discussion of the mathematical treatment.

[7] E. O. Hulburt, J. Opt. Soc. Am. **22**, 408 (1932).
[8] H. Pettersson, Göteborgs Kungl. Vetenskaps—och Vitterkets—Samhalles Handl. **B4**, 4 (1934).
[9] W. M. Powell and G. L. Clarke, J. Opt. Soc. Am. **26**, 111 (1936).
[10] W. R. G. Atkins and H. H. Poole, Phil. Trans. Roy. Soc. London **B222**, 129 (1932).

A Submarine Photometer for Studying the Distribution of Daylight in the Sea

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(Received February 7, 1938)

This paper describes the construction of a photometer which is designed to measure narrow spectral bands of submarine daylight. The photometer is capable of making rather rapid and nearly simultaneous measurements of spectral bands of both transmitted and scattered light. The design permits operation from a boat which is not provided with special electrical equipment.

SUBMARINE photometers of various designs,[1–7] using light sensitive cells, have been employed in studies of extinction coefficients of daylight in natural water bodies and in studies connected with photosynthetic processes occurring in these water bodies. For such studies, it is very desirable that the submarine photometer have certain characteristics. The extreme ends of the visible spectrum are more highly absorbed than the central portion resulting in a considerable variation in the spectral distribution of the light transmitted to various depths. The photometer should be capable of measuring the response of the light sensitive cell to a number of spectral bands of daylight at various depths. Since there is usually some variation in the content of the water column above the photometer, measurements should be made in the least time consistent with existing conditions. The photometer should be capable of making nearly simultaneous measurements of the same spectral bands of both transmitted and scattered light.

Of the photometers in use, those employed by Atkins and Poole,[1] Erikson,[4] Takenouti,[5] and Utterback and Boyle[6] have been capable of yielding information on spectral bands of submarine radiation. The Atkins and Poole photometer permits observations on but one color band during each submergence. Erikson determined extinction coefficients by exposing photographic plates to light of a definite wave-length. The photometer of Utterback and Boyle employed a filter-changing device such that eight spectral bands could be measured at various depths without bringing the instrument to the surface. None of these photometers was capable of the simultaneous measurement of transmitted and scattered light.

For the purpose of meeting the above requirements, the photometer shown in Fig. 1 was constructed.

The photometer is supported by an adjustable stainless steel sling. The upper and lower windows, of heavy plate glass, are provided with external diffusing screens of flashed opal glass. A light sensitive cell is mounted just inside of each window. The eye bolts on the upper face and the six legs on the lower face furnish the necessary protection for the diffusing screens and the windows. The photometer is connected to instruments and filter controls on shipboard through a six-conductor Tirex cable which enters the case through a water-tight gland.

The entire assembly consisting of the filter disks, the photoelectric cells and the actuating motor is mounted, as shown in Fig. 2, in such a manner that it may be placed in or withdrawn from the case as a unit. This feature of the design is of great convenience in that it permits the making of adjustments without disturbing the relative positions of various parts or the breaking of more than one of the water-tight seals. The mechanical design is such that the device will operate in any position.

The light sensitive cells, A, are mounted on specially designed ring supports which permit

[1] W. R. G. Atkins and H. H. Poole, Phil. Trans. Roy. Soc. Dublin **B478**, 129 (1933).
[2] H. H. Poole and W. R. G. Atkins, Phil. Trans. Roy. Soc. London **A235**, 1 (1935). J. Mar. Biol. Assoc. **19**, 727, 1934.
[3] Hans Pettersson, Göteborgs Kungl. Vetenskap-och Vitterhets-Samhälles Handl. **B4**, 5 (1934).
[4] Henry A. Erikson, J. Opt. Soc. Am. **23**, 170 (1933).
[5] Yositada Takenouti, Bull. Jap. Soc. Sci. Fisheries **5**, 3 (1936).
[6] C. L. Utterback and J. Watson Boyle, J. Opt. Soc. Am. **23**, 177 (1933).
[7] C. L. Utterback, Rapp. et Proc.-Ver. Conseil Int. Vol. CI, 2ème Partie (1936).

freedom of adjustment. The upper pair of cells is provided with flanges, *B*, which shield the cells from stray light. The filter disks, *C*, are mounted on a shaft which passes axially through a specially designed motor, *D*, which, by means of a ratchet, advances the disk assembly through a fixed angle. A driving pawl, *F*, is mounted on the arm, *G*, which is rigidly attached, by means of a taper pin, to the motor shaft. The throw of this arm in its forward motion is limited by the spring stop, *H*, which forces the pawl firmly into the notch of the ratchet on the filter disk hub thus preventing the filters from advancing beyond the proper position. The pawl arm is returned to its initial position by means of the helical spring, *K*. The rubber cushioned stop, *L*, limits the return motion of the arm. The detent, *M*, prevents a reverse motion of the filter assembly.

The motor is shown in detail in Fig. 3. The rotor, *A*, the pole pieces, *B*, and the case, *C*, were machined from mild steel. The bearing plates, *D*, are brass and are provided with ball bearings designed to withstand end thrust. The coil, *E*, which is mounted concentric with the rotor, has a resistance of approximately five ohms and operates the motor with a current of approximately one ampere.

The magnetic gap between the rotor and the pole pieces is only 0.004". Due to the low reluctance of the magnetic circuit the motor has a very high efficiency, thus enabling the device to be operated from a 12-volt portable dry battery or a storage battery of small capacity. This feature of design permits the operation of the photometer from a boat which is not provided with special electrical equipment.

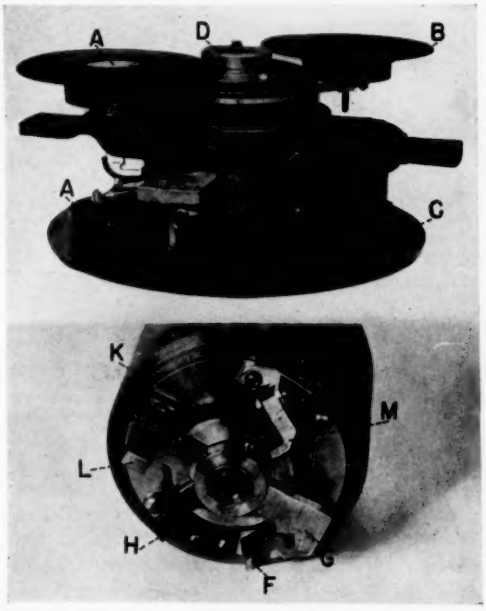

FIG. 2. Motor and filter disk assembly.

The throw of the filter disk can be altered by changing the number of notches on the ratchet, *F*, and the position of the stops. This permits the photometer to be used with a different number of filters if desired.

While any type of photoelectric cell may be used in the photometer, it has been the authors' experience that submarine light measurements are more readily made with some form of rectifier or barrier cell. Such a cell does not require an external potential and thus does away with troublesome leakage currents and insulation difficulties which arise on work at sea. Its spectral sensitivity approximates that of the eye.

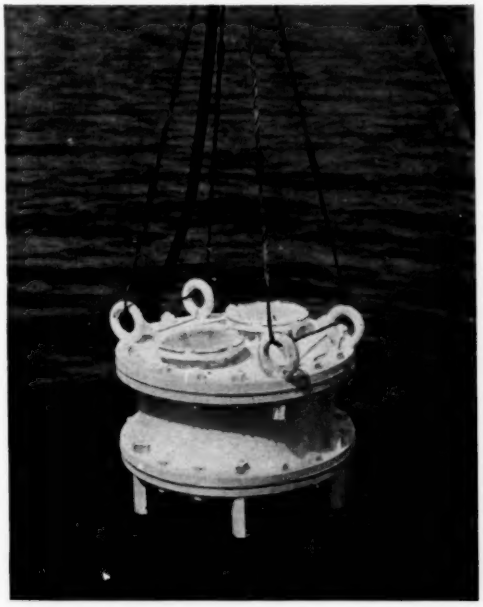

FIG. 1. Submarine photometer.

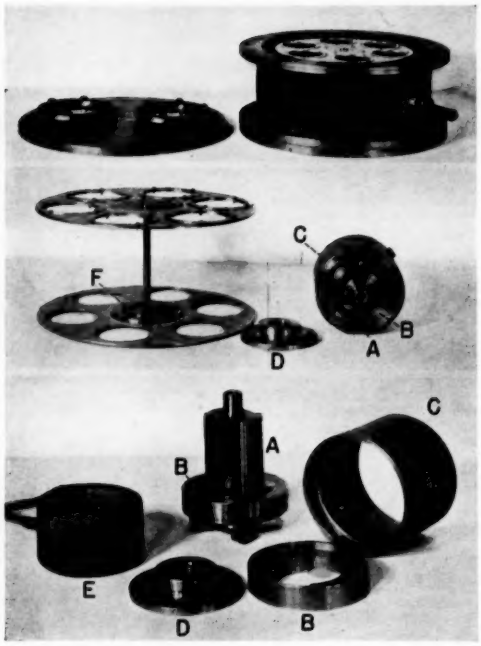

FIG. 3. Details of photometer.

Two Weston photronic cells are mounted just below the upper filter disk in such a way that they may be freely adjusted for position. These cells are exposed through the color filters to diffuse sun and sky light. Under such conditions the cells have shown but little, if any, fatigue and have maintained a constant current-illumination relation. The current output is approximately linear with illumination under the conditions of use but would depart from linearity with a high external resistance or if exposed to intense light.

Electrocells are mounted just above the lower filter disk. Since the current output of the electrocell is considerably greater than that of a photronic cell of equal area, it is preferred for measurements of the low intensity scattered light. It should not be exposed to direct unfiltered sunlight.

Leads from the light sensitive cells are brought to a terminal board within the photometer case. The conductors of the cable also terminate at the same board thus permitting flexibility of connections.

The authors have used Eastman gelatin light filters and Schott glass filters. The Schott RG 1 and RG 2 have been found to be very useful for the red end of the spectrum. They have fairly narrow transmission bands and sharp cut-offs on the short wave-length side. The Schott BG 12 has been found desirable for blue light. Several Eastman gelatin filters have given useful results. Gelatin filters should be sealed with Canada balsam between thin glass plates.

The photometer has been successfully used at sea in both rough and calm water and in the inshore waters of the Pacific Northwest. It permits a rapid examination of the transmission and scattering of fairly narrow spectral bands of solar radiation in bodies of water. The photometer is now being used in connection with studies of the physical and biological significance of submarine radiation.

One of us (C. L. U.) wishes to express his appreciation of a grant-in-aid from the National Research Council. The construction of the photometer and the purchase of the submarine cable was made possible by a grant-in-aid.

Photometric Relationships Between Complementary Colors*

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Two complementary chromaticities must be additively mixed in the proper brightness ratio in order to produce a neutral mixture. This ratio, L, can be computed from any one of the following expressions.

$$L/y+1/y'=(1+L)/y_n,$$
$$L=(y'/y)\cdot(y-y_n)/(y_n-y'),$$
$$L=(y'/y)\cdot(x-x_n)/(x_n-x').$$

The symbols, x, y, and x', y', are the coordinates of the complementary chromaticities in the I.C.I. chromaticity diagram, in which the point x_n, y_n represents the chromaticity of the desired "neutral" or gray mixture. The three points (x, y), (x_n, y_n), and (x', y') must be colinear in this order. Neither of the complementaries need be spectrum colors, but the equations may be used to determine the brightness ratios of spectrum complementaries necessary for neutral mixtures. The great brightness ratios resulting from the low luminosity of the blue end of the spectrum are inconvenient for graphical representation, and can be avoided if the energy ratios of complementary spectrum lights necessary for a neutral mixture are computed. Loci can be traced in the I.C.I. chromaticity diagram to give the "moment" of one millilambert of every chromaticity with respect to any assigned neutral stimulus. The brightness ratio of two complementary chromaticities necessary for a neutral mixture is the reciprocal of their moments per millilambert with respect to the neutral. The locus of constant moment (per millilambert), m, is a conic section having the $y=0$ axis as its directrix, the neutral point as its focus and the moment, m, as its eccentricity. This diagram is of immediate use in the plane vector solution of any problem of additive color mixture. The vector solution of the problem encountered in the projection of additive photographs, the determination of the brightness ratios of the three primaries necessary for a neutral screen, is outlined. The "neutral" chromaticity desired on the screen need not be the neutral with respect to which the moment loci are drawn. The colorimetric purity of any color is the ratio of the moment per millilambert of that color to the moment per millilambert of the spectrum color colinear with the neutral and the sample color. Formulas and tables for the interconversion of colorimetric and excitation purities are given. The dominant wave-lengths and visual efficiencies, in illuminant "C," of colors having maximum visual efficiencies are plotted as functions of the wave-length limits of the absorption bands, and the complementary relations between such colors are reviewed. Formulas are given for the determination of the chromaticities in illuminant "C" of all colors for which the spectrophometric curves are linear functions of wave-length. All such colors have the dominant wave-length 580.1 mμ, excitation purity less than 55 percent, or dominant wave-length 480.1 mμ, excitation purity less than 35 percent.

A. Definition of Complementary Color

SEVERAL alternative definitions of "complementary colors" have been proposed. Only that definition which states that the optically additive mixture of two complementary colors must match some arbitrarily assigned "neutral" stimulus is sufficiently specific to permit the solution of the general problem of the colorimetric specification of all of the possible complementaries of any given color.

Special merits are claimed for other definitions of complementary colors. For instance, it is occasionally claimed that the color of the after-image produced by fixation on a given color followed by observation of an equally bright neutral stimulus is a better aesthetic complementary than any of the additive complementaries of the given color. Since there are few data available concerning these "after-image complementaries," and since there is evidence that these after-image colors vary continually in hue as well as in saturation,[1] there is no apparent way in which the after-image or "fatigue complementary" of any given color can be identified unambiguously. Such identification seems to be a prerequisite to any serious study of the harmonious relationship which is said to exist between the after-image "complementaries." In some discussions, the after-image colors are assumed, either implicitly or explicitly, to be identical with the additive complementaries. This assumption is unsupported by any experimental data with which the present author is acquainted, and appears to have originated in an over-simplified conception of the Young-Helmholtz theory of color vision. Other older definitions of complementary colors, based on the behavior of subtractive mixture of colorants, are

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rather generally recognized to be untenable, since they originate in fortuitous phenomena, governed by the spectrophotometric characteristics of the colorants, having little or no physiological or psychological significance, and failing even more seriously than the after-image phenomenon to identify "complementary" colors in any unambiguous manner. Doubtless, a thorough survey of the literatures of art and of psychology would reveal other definitions of complementary colors, but, so far as the present author is aware, none of these definitions permits the unambiguous identification of the colors complementary to any given color.

In the interests of clarity, the additive mixtures discussed in this paper will be considered to be the superposition of two images on a perfectly reflecting and perfectly diffusing screen. These images exhibit separately the brightnesses and chromaticities described for each case. When spectrophotometric characteristics are specified, these can be thought of as the spectral transmittance of light filters placed in the beams forming the images. The light source is assumed to have the spectral energy distribution of the I.C.I. illuminant "C."[2, 3] A spinning disk (Maxwell disk, or color top) having equal (180°) segments of the two specified colors, consisting of opaque surfaces having the specified colorimetric or spectrophotometric characteristics, and illuminated with the I.C.I. illuminant "C," would be colorimetrically equivalent to the projection screen method. In the case of the composite image on the screen, the tristimulus values (and brightness) of the mixture are the sums of the corresponding tristimulus values of the two superimposed images.[3] In the case of the spinning disk, these tristimulus values are the averages of the corresponding tristimulus values of the two equal segments. The color of the screen with any intensity of "C" illumination will be taken arbitrarily as the neutral stimulus throughout this discussion.

It is generally known that the additive mixture of any color with a suitable brightness of any other color will produce a neutral mixture, provided that the point representing a neutral stimulus in the chromaticity diagram lies on the straight line drawn between the points representing the two colors. Such pairs of colors are complementary by the definition adopted in the first paragraph. It follows that there are an infinite number of colors complementary to any given color. Of these complementary colors, there is at most only one chromaticity having any assigned brightness. Conversely, any one of these complementary chromaticities is additively complementary to the assigned color in only one brightness. Two colors will be considered complementary colors only if they have the correct relative brightnesses as well as the correct chromaticities necessary to produce a neutral additive mixture in the projection screen experiment. Colors which have suitable chromaticities but incorrect brightness ratios will be said to have complementary chromaticities, but will not be called complementary colors.

B. Photometric Relationships Between Complementary Colors

Figure 1 permits the identification of the complementary color having its brightness in any given ratio to the brightness of the original color. Conversely, it can be used to determine the brightness ratio of any pair of complementary chromaticities necessary for a neutral additive mixture. The ratio of brightnesses necessary for a neutral additive mixture of any two chromaticities colinear with the neutral point ($x=0.3101$, $y=0.3163$) is the inverse of the ratio of the moments of one millilambert of the respective chromaticities, indicated by the loci in Fig. 1. For instance, one millilambert of a color having dominant wave-length 570 mμ and excitation purity 72 percent ($x=0.4067$, $y=0.4882$) has a moment of 0.40 with respect to the I.C.I. illuminant "C." The complementary color having the same brightness must have the same moment per millilambert with respect to illuminant "C." Consequently, it must be represented at the intersection of the extended 570 mμ dominant wave-length line with the locus of moment 0.40. This intersection is at $x=0.2880$, $y=0.2775$, and the corresponding color has the dominant wave-length 457.9 mμ and 13.4 percent excitation purity. The color having dominant wave-length 457.9 mμ and 45 percent excitation purity ($x=0.2370$, $y=0.1862$) has a moment of 0.80 per millilambert. Consequently, this color should

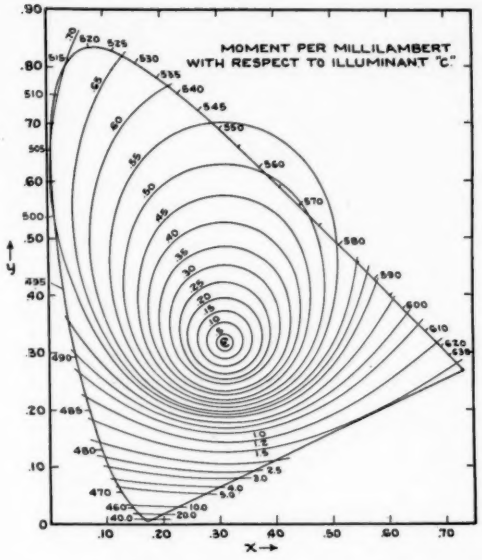

FIG. 1. Moment diagram, showing moment of one millilambert of any color, with respect to the I.C.I. illuminant "C" as neutral. Ratio of brightnesses of two complementary colors is the inverse of the ratio of the indicated moments per millilambert.

be only one-half as bright as the complementary yellow having 72 percent excitation purity, in order to result in a neutral additive mixture. The colorimetric purity of any color is the ratio of the moment per millilambert of that color to the moment per millilambert of the spectrum color colinear with the neutral and the sample color.

Figure 1 is easily constructed by making use of the fact that for the purposes of color computation with the aid of the center of gravity principle, the "mass" of any color in the I.C.I. chromaticity diagram is the brightness divided by the y coordinate.[3, 4] The moment of any color with respect to illuminant "C" is the product of this mass times the distance in the I.C.I. diagram from the point representing the color to the point representing illuminant "C." For one millilambert brightness of any color, this moment is therefore defined as the ratio, A/B, of the distances: A, from the sample point to the neutral point; and B, from the sample point to the horizontal (x) axis of the I.C.I. diagram. Therefore, each locus of any assigned value of the moment per millilambert, m, shown in Fig. 1 is a conic section having the neutral point as its focus, the x axis as its directrix, and the assigned moment per millilambert, m, as its eccentricity.

Since, in order to produce a neutral mixture, two colors must be colinear with the neutral point, and must have equal moments with respect to illuminant "C," their brightnesses must be in inverse proportion to their moments per millilambert, as indicated in the figure. If x, y, are the coordinates of the point representing one color, if x', y' are the coordinates of some color complementary to the first color, and if L represents the ratio of the brightness of the first color as compared with that of the second, then it can be shown that:

$$L=(y'/y)\cdot(y-0.3163)/(0.3163-y') \quad (1a)$$

or $$L=(y'/y)\cdot(x-0.3101)/(0.3101-x') \quad (1b)$$

or $$L/y+1/y'=(L+1)/0.3163. \quad (2)$$

These equations can be used in place of Fig. 1 for the solution of the problems discussed above. Eqs. (1a) and (2) should not be used when the dominant wave-length of either of the complementaries is between 485 mμ and 495 mμ, because the differences of the y coordinates of such colors are subject to relatively great rejection errors. Eq. (1b) is designed to handle these cases. It is essential to remember that the points (x, y) and (x', y') must represent complementary chromaticities, that is, they must be colinear with the neutral point ($x=0.3101$, $y=0.3163$).

Figure 2, showing the brightness of the spectrum complementary necessary for a neutral mixture with unit brightness of each spectrum color[5–8] can be determined immediately from Fig. 1. The values shown by the curves in Fig. 2 can be computed more accurately from Eqs. (1a) and (1b) by substituting the values of the coordinates of spectrum colors for every pair of complementary wave-lengths, given in Tables XXIV and VIII, respectively, in the *Handbook of Colorimetry*.[3]

Curve (a) in Fig. 3 shows the moment per millilambert of each spectrum color. This curve can be determined directly from Fig. 1. This curve resembles the curve given by Sinden in his Fig. 3[6] and named by Sinden, in accordance

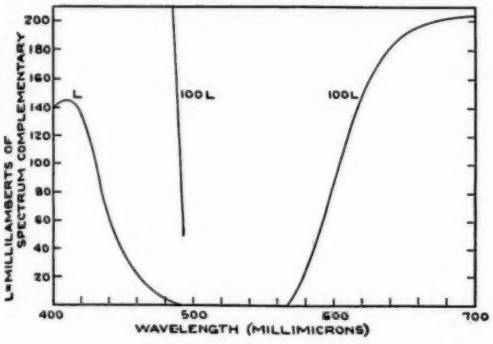

FIG. 2. Brightness of complementary wave-length necessary to "neutralize" one millilambert of indicated wave-length.

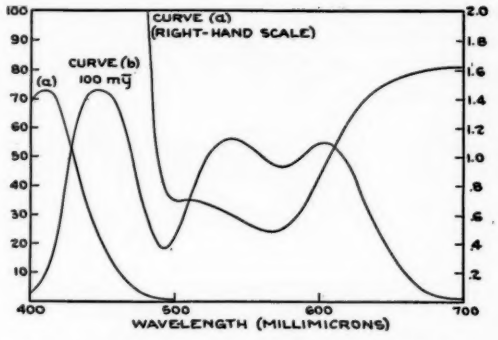

FIG. 3. (a) Moment of one millilambert of indicated wave-length. (b) Moment of one watt of indicated wave-length.

with the suggestions of Helmholtz[5] and others,[8] "relative subjective saturation of the spectral hues." This curve is shown by the present derivation to be a consequence of the objectively verifiable laws of additive color mixture, and no support for the claim of subjective significance can be found in this derivation. Curve (b) shows the relative moment per watt of each spectrum color, determined by multiplying each ordinate of curve (a) by the corresponding value of the luminosity[9] function, $\bar{y}$. The inconveniently high values shown by curve (a) for the short wave-lengths are thus eliminated, by placing the data on an energy basis.

Figure 4 shows the number of watts of the spectrum complementary necessary for a neutral mixture with one watt of each wave-length of the spectrum. This curve was derived by multiplying each ordinate of the curve in Fig. 2 by the ratio of the value of the luminosity function at the indicated wave-length to the value of the luminosity function at the complementary wave-length. This eliminates the excessively high values shown by the curve in Fig. 2 in the blue end of the spectrum. By thus placing the data on an energy basis, the engineering implications of the curve are more clearly shown. For instance, it is seen that, aside from the wave-lengths between 492 mμ and 567 mμ, which have either extreme red or extreme violet complementaries or no spectrum complementaries whatever, the orange at 605 mμ is the most efficient spectrum color in mixtures with its complementary. This orange spectrum color is almost twice as effective in neutralizing one watt of its blue-green spectrum complementary (490 mμ) as is the most effective blue (448 mμ) in neutralizing one watt of its yellow-green complementary (569 mμ).

C. Vector Solution of Problems of Additive Color Mixture

Figure 1 can be used to advantage in any problem in which a neutral additive mixture is to be established. If, for instance, it is necessary to secure a white image on the screen in connection with an additive method of projection of colored photographs, suitable relative brightnesses of the three projection primaries can be readily determined. Three vectors should be drawn, through the illuminant point and through the points representing the chromaticities of the three projection primaries. One of these vectors should be assigned a length equal to the moment of one millilambert of the corresponding chromaticity, as given by Fig. 1. A second of the vectors must then be given such a length that the resultant of these first two vectors will be parallel but oppositely directed to the third vector. The length of the second vector necessary for this adjustment can then be divided by the moment per millilambert of the second projection primary (determined from Fig. 1), to give as a result the proper brightness ratio of the second to the first projection primary. The chromaticity of the resultant mixture will be represented by the intersection of this resultant vector and the line joining the points representing the chromaticities of the first two pro-

jection primaries. The moment of this mixture, per millilambert of the first component only, is represented by the length of the resultant vector. Therefore, the moment of the third projection primary necessary to neutralize this additive mixture of the first two primaries is represented by the equally long but oppositely directed vector through the illuminant point and the point representing the chromaticity of the third primary. The length of this resultant vector can therefore be divided by the moment per millilambert of the third primary (determined as previously from Fig. 1) to give the required brightness ratio of the third to the first projection primaries.

The construction mentioned above, in which the chromaticity and moment of the additive mixture of any two colors can be determined is of use in all problems of additive mixture, and the moments with respect to illuminant "C" shown in Fig. 1 can be used for this construction even when illuminant "C" has no relation to the problem. The vectors must, however, be drawn through the point representing illuminant "C," if the moments from Fig. 1 are used. Any desired neutral point can be introduced as a new color having a chromaticity and moment with respect to illuminant "C."

Figure 5 illustrates the general method. The problem is to determine, using Fig. 1, the brightness ratios of the three projection primaries, R, G, and B, necessary to produce an image on the screen having the chromaticity of the I.C.I. illuminant "A," represented by the point, A, in Fig. 5. The chromaticity of the illuminant "C," the reference point for all vectors and moments, is represented by the point, C, in Fig. 5. Construct the vector, g, from the point, C, through the point, G, and having a length equal to the moment specified for the point, G, by the loci in Fig. 1. Any convenient scale of vector length will be satisfactory, if used consistently. The point, F, at the intersection of the lines, GB and RA, represents the chromaticity of the mixture of the primaries, B and G, necessary for a mixture with the primary, R, in order to match illuminant "A." Construct the vector, f, from the point, C, through the point, F, and the vector, b, from C through B. The vectors f and b are of such a length that f is the resultant of b and g as shown by the dotted parallelogram construction. The length of b can be divided by the moment shown for the point, B, in Fig. 1, to give the desired ratio of the brightness of the primary, B, to the brightness of the primary, G. The vectors, r and

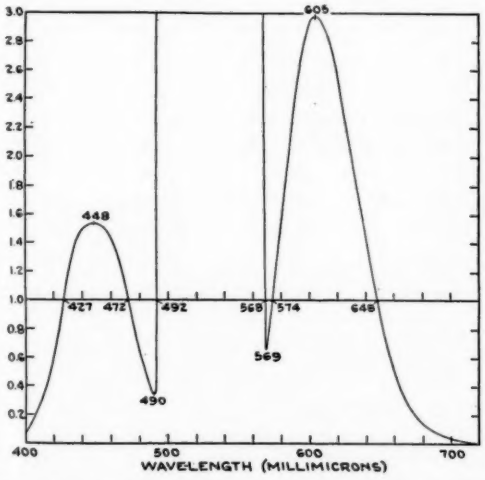

FIG. 4. Watts of complementary spectrum light necessary to "neutralize" one watt of indicated wave-length.

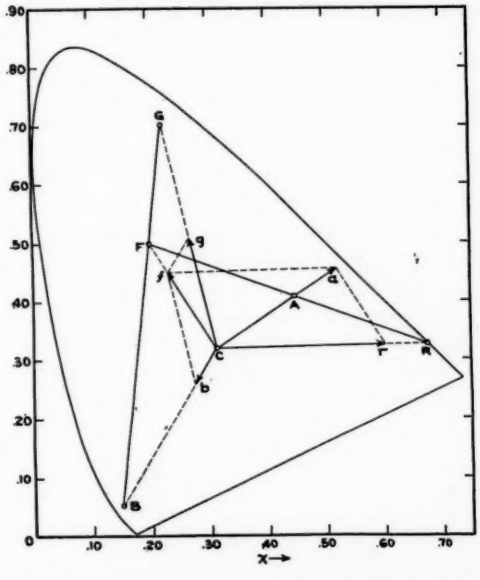

FIG. 5. Typical vector solution of a problem of additive synthesis of a desired color from three known primary chromaticities.

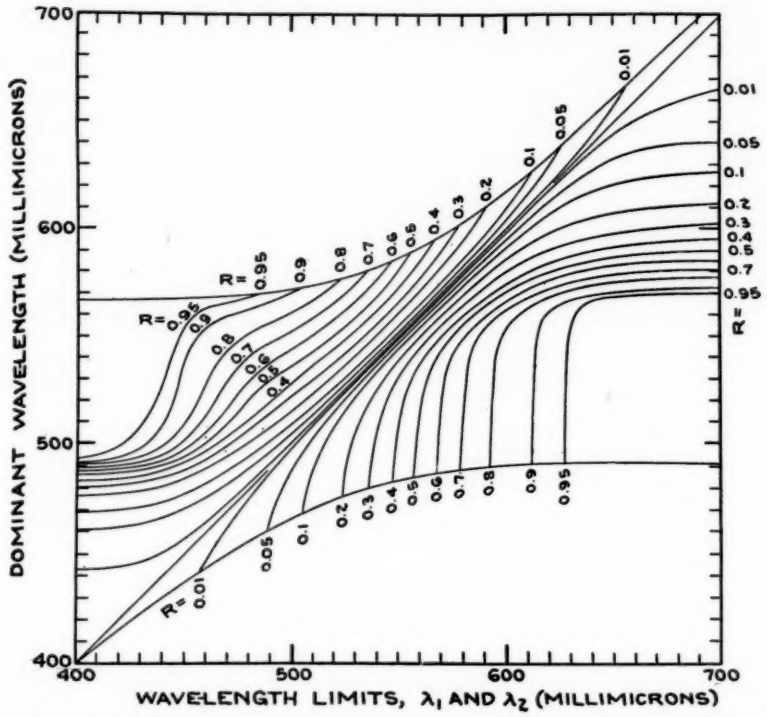

FIG. 6. Dominant wave-lengths of colors of maximum visual efficiency as functions of visual efficiency and of short or long wave-length limits of transmission or reflection band.

a, are now constructed from the point, C, through the points, R and A, respectively, and of such lengths that a is the resultant of f and r, as shown by the parallelogram construction. The length of r can be divided by the moment shown for the point, R, in Fig. 1, to give the ratio of the brightness of the primary, R, to the brightness of the primary, G. The length of the vector, a, can be divided by the moment shown for the point, A, in Fig. 1 to give the ratio of the brightness of the mixture of the three primaries to the brightness of the green component. This ratio furnishes a convenient check on the calculations.

D. Complementary Relations Between Colors Having Maximum Visual Efficiencies

The discussion thus far has been quite general in applicability, the properties of additive mixtures being independent of the spectroradiometric characteristics of the components of the mixtures. There is some interest in the complementary relationships between colors having maximum visual efficiencies.[10] From the definition of such colors it follows immediately that the brightnesses of two physically complementary[3] colors having maximum visual efficiencies must add up to the brightness of a perfect reflector in the same illumination. Very little else can be said about the complementary properties of these colors of maximum visual efficiency which has not been discussed above as general properties of all complementary colors. There is some interest, however, in the following data which have been plotted from Table II of the author's earlier paper on "Maximum Visual Efficiencies of Colored Materials."[10–15] Additional data, not contained in this table, were necessary for the completion of the curves in Fig. 6, and these data were computed according to form A of the above-mentioned article.

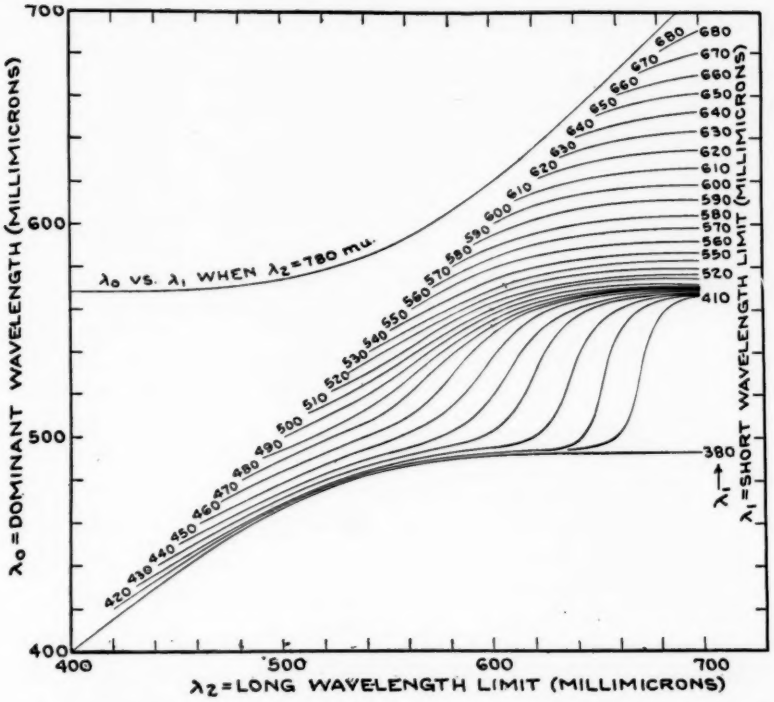

FIG. 7. Dominant wave-lengths of colors of maximum visual efficiency as functions of short and long wave-length limits of transmission or reflection band.

Figure 6 shows, for the indicated values of visual efficiency, the dominant wave-lengths of colors having complete absorption for all wave-lengths short of λ_1, complete transmission or reflection from λ_1 to λ_2, and complete absorption of all wave-lengths longer than λ_2. Conversely, Fig. 6 shows the values of λ_1 and λ_2 necessary for a color having the indicated dominant wave-length and visual efficiency. If the values of λ_1 and λ_2 necessary for a color having maximum visual efficiency and an assigned dominant wave-length and purity are to be determined, the corresponding value of the visual efficiency should be determined from Fig. 3 of the article on "Maximum Visual Efficiency of Colored Materials."[10] The values of λ_1 and λ_2 can then be determined from Fig. 6 of this report, as functions of the dominant wave-length and visual efficiency. Whenever the chromaticity to be investigated is on the purple side (toward the x axis) of the sharp ridges indicated in Figs. 3 and 4 of the article on "Maximum Visual Efficiency,"[10] the corresponding spectrophotometric curves are in class (b), typified by the solid line of Fig. 1b of the same article. For these colors, all wave-lengths of the visible spectrum are completely reflected or transmitted except those between λ_1 and λ_2, which are completely absorbed. In such cases, it is necessary to determine λ_1 and λ_2 for the physically complementary color. The dominant wave-length of this physically complementary color can be determined from the chromaticity of the desired color by the use of Table VIII and the chromaticity diagrams in the *Handbook of Colorimetry*,[3] or by computation and interpolation from D. B. Judd's[16] Table VI. Judd's table shows the slope (or the reciprocal of the slope) of the line from the point representing illuminant "C" to any sample point as a function of the dominant or complementary wave-length. The visual efficiency of the physical complementary color will be

$$R' = 1 - R, \tag{3}$$

where R is the visual efficiency of the color transmitting or reflecting both extremes of the spectrum. The values of λ_1 and λ_2, the limits of the absorption band, can be determined from Fig. 6 as soon as the complementary wave-length and R' are known. Similarly, the converse problem of determining the chromaticity and visual efficiency of any color of maximum visual efficiency having absorption between the wave-lengths λ_1 and λ_2 can be determined as the complementary of the color which transmits only between these two limits.

Figure 7 has been constructed from Fig. 6 by holding λ_1 fixed and determining the value of λ_2 corresponding to many pairs of values of the dominant wave-length and visual efficiency. The values of the corresponding values of visual efficiency are omitted from Fig. 7 because these can be determined from Fig. 6. Fig. 7 serves primarily to determine the dominant wave-length of any color transmitting or reflecting completely all wave-lengths between λ_1 and λ_2 and absorbing all wave-lengths outside of these limits. As in the case of Fig. 6, it is necessary to determine the dominant wave-length of the complementary whenever any problem concerns maximum efficiency colors transmitting or reflecting both extremes of the visible spectrum. After the dominant wave-length corresponding to any given values of λ_1 and λ_2 has been determined from Fig. 7, the value of the visual efficiency can be determined from Fig. 6, and the purity from Fig. 3 of the article on "Maximum Visual Efficiency of Colored Materials."[10]

TABLE I. *For spectral hues.*

Dominant Wave-length (in mμ)	a	b	Dominant Wave-length (in mμ)	a	b	Dominant Wave-length (in mμ)	a	b
380	0.016	−0.984	500	1.702	0.702	590	1.338	0.338
390	0.0155	−0.9845	505	2.068	1.068	595	1.253	0.253
400	0.015	−0.985	510	2.374	1.374	600	1.178	0.178
410	0.015	−0.985	515	2.567	1.567	605	1.110	0.110
420	0.016	−0.984	520	2.654	1.654	610	1.057	0.057
430	0.022	−0.978	525	2.613	1.613	615	1.010	0.010
440	0.034	−0.966	530	2.546	1.546	620	0.974	−0.026
450	0.056	−0.944	535	2.473	1.473	625	0.946	−0.054
455	0.072	−0.928	540	2.383	1.383	630	0.924	−0.076
460	0.094	−0.906	545	2.290	1.290	635	0.904	−0.096
465	0.126	−0.874	550	2.193	1.193	640	0.889	−0.111
470	0.183	−0.817	555	2.083	1.083	645	0.876	−0.124
475	0.275	−0.725	560	1.973	0.973	650	0.867	−0.133
480	0.420	−0.580	563	1.908	0.908	660	0.855	−0.145
485	0.645	−0.355	565	1.864	0.864	670	0.848	−0.152
490	0.933	−0.067	567	1.821	0.821	680	0.844	−0.156
492	1.074	0.074	570	1.723	0.723	690	0.840	−0.160
493	1.151	0.151	575	1.645	0.645	700	0.839	−0.161
495	1.307	0.307	580	1.538	0.538	780	0.839	−0.161
497	1.461	0.461	585	1.434	0.434			

E. Formulas and Tables for Interconversion of Colorimetric and Excitation Purities

It is occasionally necessary in these and other problems to convert data from excitation purity[3] to colorimetric purity,[16, 17] or to make the reverse transformation. This transformation can be made graphically by the use of Fig. 1, using the rule given at the end of the first paragraph of Section B. The fundamental algebraic formulas for this transformation are given on page 60 of the *Handbook of Colorimetry*[3] but the following simplified formulas and tables of parameters for illuminant "C" are very convenient. The formula for the conversion from excitation purity, p_e, to colorimetric purity, p_c, is:

$$p_e = a \cdot p_e / (1 + b \cdot p_e). \tag{4}$$

The formula for the reverse transformation is:

$$p_e = p_c / (a - b \cdot p_c). \tag{5}$$

For spectrum hues the parameter, a, is given by

$$a = y_\lambda / y_w, \tag{6}$$

where y_λ is the y coordinate of the spectrum color having the dominant wave-length of the sample and where y_w is the y coordinate of the neutral point. For spectrum hues, the parameter, b, is:

$$b = a - 1.0. \tag{7}$$

The values of the parameters, a and b, for spectrum hues and for the I.C.I. illuminant "C" taken as the neutral are given by Table I.

TABLE II. *For purples.*

Complementary Wave-length (in mμ)	a	b
492	−1.808	−0.124
493	−1.601	−0.210
495	−1.402	−0.330
497	−1.295	−0.408
500	−1.169	−0.483
505	−1.069	−0.552
510	−1.030	−0.596
515	−1.027	−0.625
520	−1.040	−0.649
525	−1.082	−0.669
530	−1.127	−0.684
535	−1.178	−0.703
540	−1.248	−0.723
545	−1.329	−0.748
550	−1.427	−0.777
555	−1.571	−0.816
560	−1.758	−0.866
563	−1.908	−0.908
565	−2.033	−0.942
567	−2.183	−0.985

For purples:

$$a=(y_\lambda/y_w)\cdot(y_{\lambda'}-y_w)/(y_\lambda-y_w), \qquad (8)$$

where y_λ and y_w are defined as above, and $y_{\lambda'}$ is the y coordinate of the intersection of the line drawn between the extremities of the spectrum locus with the straight line through the sample and the neutral points. The parameter, b, for the purples is given by

$$b=(y_{\lambda'}-y_w)/y_w. \qquad (9)$$

Table II gives the values of the parameters, a and b, for purple hues and for the I.C.I. illuminant "C," as functions of the complementary wave-lengths.

F. Linear Spectrophotometric Curves

A brief mention of a subject different from, but not wholly unrelated to, the topic of complementary colors will probably not be too much out of place here. Many spectrophotometric curves of naturally occurring substances exhibit an almost linear relationship between reflectance (or transmittance) and wave-length. This characterizes most of the spectrophotometric curves for human hair,[18] in addition to many of those for wood, paper pulp, and untinted papers, textiles, and minerals. It has been noticed frequently that such samples have dominant wave-lengths in the neighborhood of 575 mμ to 580 mμ. If the reflectance is assumed to bear a strictly linear relationship to the wave-length, within the visible spectrum, then it can be represented by the formula:

$$R_\lambda=c+d\cdot(\lambda-400\ \text{m}\mu)/(300\ \text{m}\mu) \qquad (10)$$

between the wave-lengths 400 mμ and 700 mμ. In this formula, the constant, c, is the value of the reflectance, R_λ, at 400 mμ and d is the difference between the values of the reflectances at 700 mμ and 400 mμ. Computation for the case of $c=0$ and $d=1$, with illuminant "C," results in the tristimulus values:

$$X=0.5400,\ Y=0.5243,\ Z=0.2172,$$

corresponding to $x=0.4220$, $y=0.4095$, dominant wave-length 580.1 mμ and excitation purity 54.9 percent.

In general, it can be shown that:

$$X=0.9804c+0.5400d,$$

$$Y=\quad c+0.5243d,$$

$$Z=1.1812c+0.2172d,$$

$$x=\frac{0.9804c+0.5400c}{3.1616c+1.2815d},$$

$$y=\frac{c+0.5243d}{3.1616c+1.2815d}.$$

Dominant wave-length = 580.1 mμ when d is greater than zero.

Dominant wave-length = 480.1 mμ when d is less than zero. In the extreme case for which $c=1.0$ and $d=-1.0$, the dominant wave-length is of course 480.1 mμ, the excitation purity is 34.8 percent, and the brightness (Y) relative to magnesium oxide (a practically perfect diffuse reflector) in the same illumination is 47.6 percent.

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The Design of Monochromatic Filters for the Type H-3 Mercury Lamp

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The present popularity of the type H-3 mercury lamp justifies a further consideration of mercury monochromats. The design of mercury monochromats represents a choice of glasses and thicknesses to achieve the highest purity (ratio of transmission at the designed wave-length to total transmission) permitted by the energy distribution of the source and the spectral response of the receiver. The filter glasses (usually two) in the monochromat are selected by considering the steepness of slope of their transmission curves near the designed wave-length and their degree of opacity in adjoining regions. The proportionate thicknesses can then be varied for the most favorable values of transmission at the desired wave-length and of purity. The best thickness ratio between the components is found by computing the purities of several combinations whose total thicknesses are adjusted to give constant transmission at the designed wave-length. It is assumed that the best thickness ratio is practically independent of the thickness of the total filter over the range investigated. By varying the total thickness, keeping the ratio constant, a relation between the transmission at the designed wave-length and the purity will be shown. Monochromats have been designed for use with the type H-3 mercury lamp utilizing caesium and sodium surface photo-tubes, the selenium surface barrier-layer cell, the thermopile and the eye as detectors. Curves will be given showing the best values of purity and transmission at the desired wave-length for the various source-filter-receiver combinations.

MERCURY monochromats have been described at various times in the literature and in manufacturers' catalogs. However, no attempt has heretofore been made to correlate the transmission of the filter and the spectral purity of the energy transmitted. These two properties are closely related and their determination is governed by the conditions of the particular problem for which the monochromat is designed.

Fifteen different monochromats were designed to secure the best possible filter-receiver combinations for use with the type H-3 mercury lamp having high purity factors for maximum transmissions at the wave-lengths of design. There is no one method of design which can be generally adopted; the selection of the most suitable method is controlled by several factors. Of first consideration is the purpose for which the monochromat is to be designed. The energy distribution of the light source, the spectral sensitivity of the receiver, the spectral characteristics of the filter glasses available, the degree of purity of the transmitted energy which is required and the desired or permissible transmission at the wave-length of design are all interdependent. The most efficient design implies a careful selection of these which at best is a compromise governed by the stipulations of the original problem.

The uses of mercury monochromats are numerous and varied. Their use in filter radiometry is for many purposes convenient and satisfactory. The procedure is reduced to its simplest form and when highly monochromatic filters are employed the results obtained will be relatively precise. This method obviates the need for expensive delicate equipment.

A field of research which is far from new but increasingly widespread in its applications is fluorescence microscopy. Perhaps its greatest value is in biological and medical research, but its use in the textile and paper industries, in the study of food and drugs, of oils, paint, rubber, and in research chemistry is unlimited.[1] The most efficient filter for fluorescence has a high transmission for radiation in the ultraviolet and at the same time is opaque to radiation in the visible region of the spectrum. Even a trace of radiation of wave-lengths longer than 400 mμ would dilute the fluorescent light.

The importance of colorimetric analyses in the study of sugar for commercial and technological purposes is well known.[2] The knowledge of the color qualities is of great usefulness in the study of the various methods of manufacture. For these purposes monochromatic filters iso-

[1] Rolland Jenkins, "Microscopy with Fluorescent Light," Stain Technology **12**, 167 (1937).
[2] H. H. Peters and F. P. Phelps, "Color in the Sugar Industry," Technologic Paper No. 338, National Bureau of Standards, 1927.

lating the mercury lines, particularly the yellow, green and blue lines are useful for routine investigation.

Colorimetric methods applied to the study and analysis of organic and inorganic substances are of increasing importance in industrial and chemical research and are routine procedures in laboratory work. Most colorimeters utilize either visual or photoelectric methods, in particular photo-cells of the barrier-layer type. The simplicity of the operation of these colorimeters has added much to their popularity. The sensitivity of a colorimeter can be increased many times by the use of narrow band filters of high purity. The extent to which colorimetric determinations obey Beer's law is largely dependent on the purity or monochromatism of the incident light. Monochromatic colorimetry also affords a means for investigating samples which have been diluted in colored solvents.

Other important applications of monochromats are in refractometry, in photometry, in the study of optical systems by monochromatic light, in polarimetry, in abridged spectrophotometry. Their great usefulness can be attributed to the convenience and facility of their use, and their relative inexpensiveness. The significance of monochromatic filters is emphasized, and the advantages and number and variety of their applications are greatly augmented by the high transmissions at the effective wave-lengths and the large degrees of monochromatism or spectral purity of the transmitted radiation which can be had.

The purity of the transmitted energy is a measure of the degree to which the unwanted wave-lengths have been eliminated by the monochromat. It is calculated from the equation

$$P = E_M S_M T_M / \Sigma E_\lambda S_\lambda T_\lambda,$$

where the subscript M indicates the wave-length the filter is designed to transmit and the numerator is the product of the radiant energy "E_M" of the source at the designed wave-length, the sensitivity "S_M" of the receiver and the transmission "T_M" of the filter at this wave length. The denominator is the response of the detector to all of the radiant energy of the source which is transmitted by the filter.

When the total thickness of a filter is varied a corresponding increase or decrease in purity together with a corresponding decrease or increase in T_M follow. The data for a monochromat designed for wave-length 365 mμ have been plotted to illustrate this relation and are shown in Fig. 1. For a particular application it may be required that T_M be kept at a maximum, or the purity of the transmitted energy may be the controlling factor. Again, it may be necessary to maintain both T_M and purity at the highest possible values. Since a high factor for the one corresponds to a low factor for the other a compromise must be established when there is no predetermined value for either T_M or purity with which the monochromat must comply.

In the design of highly efficient monochromats the spectral energy distribution of the source and the spectral sensitivity of the receiver must be taken into consideration. The high intensity of the type H-3 mercury lamp and the unique form of the light source have made this lamp adaptable to many problems, and because of this great usefulness the lamp was chosen for the purposes of this discussion. Due to the high pressure at which the lamp operates the lines are considerably broadened and the space between the lines

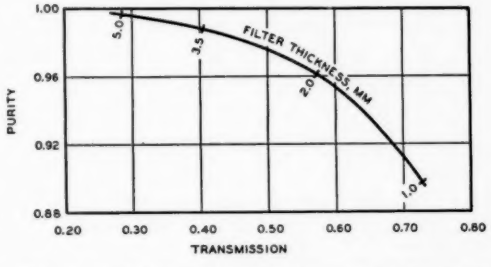

FIG. 1. Variation of transmission and purity of the transmitted energy of a filter as a function of the thickness. Data are for a monochromat designed for wave-length 365 mμ for use with a G. E. type PJ-22 photo-tube.

is occupied by a continuous spectrum. Nevertheless, intensity data for the lamp have been published in such a form that the intensity of the principal lines has been given together with the total intensities found in each of several wave bands having such boundaries that all of the energy within the bands has been considered.[3] In order to simplify the problem and make the

[3] B. T. Barnes and W. E. Forsythe, "Characteristics of Some New Mercury Arc Lamps," J. O. S. A. **27**, 83 (1937).

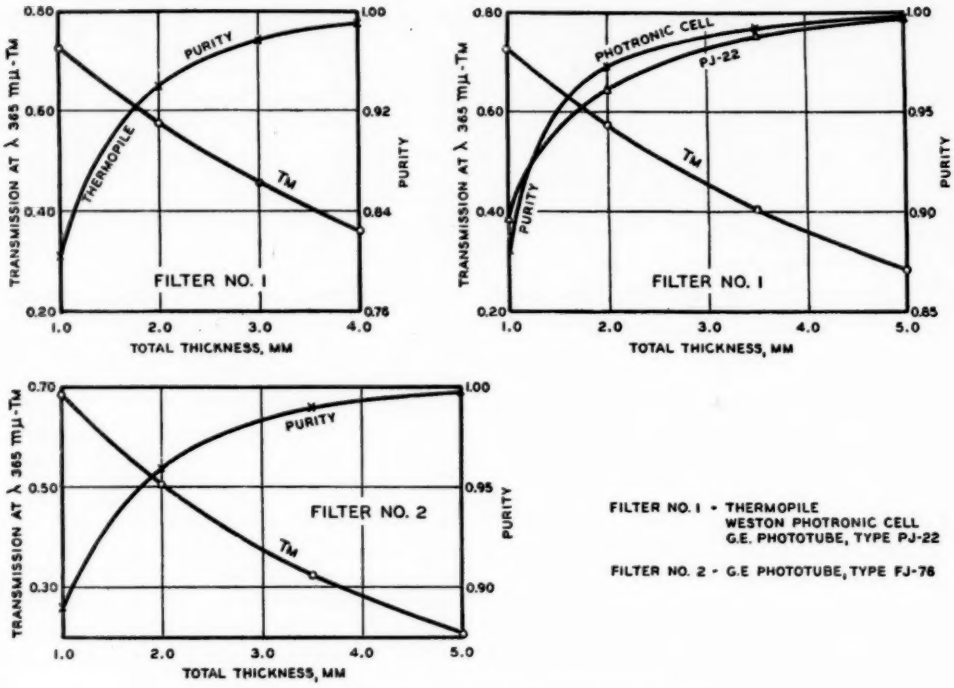

FIG. 2. Monochromats designed for wave-length 365 mμ.

calculations more convenient single wave-lengths were assigned to each band by weighting the relative intensities of the lines included in the wave bands given in the data. Where no strong lines are present the average of the band was used. These data are shown in Table I.

TABLE I. *Spectral intensities for type* H-3 *mercury lamp.*

WAVE-LENGTH, mμ	EQUIVALENT WAVE-LENGTH, mμ	SPECTRAL INTENSITY* (μw/cm² at 1 meter)
1710–1690	1700	6.9
1530	1530	6.1
1395–1357	1376	8.5
1210–1190	1200	6.3
1129	1129	7.8
1014	1014	10.8
742–683	703	3.75
683–635	659	3.0
635–595	611	3.1
595–561.5	578	19.2
561.5–532	546	33.5
532–506	519	2.2
506–484	495	2.05
484–465	475	2.2
465–447	456	2.6
435.8	436	27.1
404.7	405	14.5
366–365	365	25.7
334	334	3.2
313	313	6.9
302	302	1.21
297	297	0.73

* B. T. Barnes and W. E. Forsythe, J.O.S.A. 27, 83 (1937).

Filters were designed for use with the thermopile and with the following selective receivers: the eye, the G. E. type PJ-22 (or PJ-23) caesium-in-glass photo-tube, the G. E. type FJ-76 sodium-in-quartz photo-tube and the Weston photronic selenium surface barrier-layer cell. The standard visibility function and the relative spectral sensitivities for each photo-cell were obtained from published data.[4] These values are sufficiently accurate for the object of this paper which is to describe the methods for design, but for highest precision in a given application the characteristics of the particular lamp and receiver to be used must first be determined.

For convenience in computation the product of the energy distribution of the lamp and the

[4] G. E. Technical Information, sheet GET-480; G. E. Technical Information, sheet GET-477B; W. N. Goodwin, Jr., *Technical Data Weston Photronic Cell*, revised Sept. 1, 1934. Parry Moon, *Scientific Basis of Illuminating Engineering* (McGraw-Hill Book Co., 1936), p. 52.

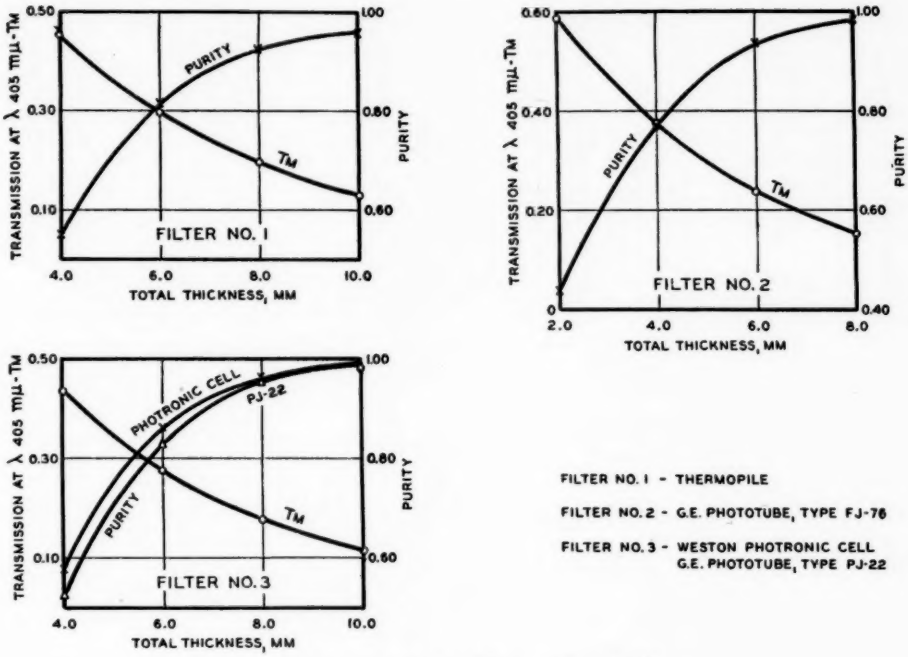

FIG. 3. Monochromats designed for wave-length 405 mμ.

sensitivities of the receivers was first secured for each wave-length and the values obtained were used as single factors. Monochromats for use with a thermopile assumed the use of a water cell having a thickness of 1 cm. This gives a cut-off in the infra-red at about wave-length 1400 mμ, which is the upper limit of the range investigated.

Most monochromats require two components, one having a sharp cut-off at the short wave-length side of the designed wave-length and the other having a sharp cut-off at the long wave-length side. These conponents were designated by *I* and *II*, respectively. The selection of appropriate component filter glasses was made by inspection. The complete spectral character of the components must be known as their opacity at wave-lengths adjoining the designed wave-length is a determining factor. In many instances the short wave-length cut-off filter has a relatively steep slope, and a few simple computations readily served to determine the most suitable thickness for this component. The purity of the transmitted energy was then computed for the filter combination for a constant thickness of component *I* and a variable thickness of component *II*. The transmissions for the range of total thicknesses investigated were equated to a constant value for T_M. The purities were plotted against the thickness ratio, *R*, where *R* is the ratio of the thickness of component *I* to the thickness of component *II*. The value of *R* which corresponded with the maximum purity was selected for the design. The value of *R* corresponding to maximum purity is not entirely independent of the total thickness. Therefore, thicknesses near the middle of the working range were used in investigating *R* and it is assumed that the best thickness ratio will hold for all useful values of the filter and is sufficiently independent of the total thickness. At the selected thickness ratio purities were computed for a practical range of total thicknesses for the combination. From these data curves were plotted for transmissions at the designed wave-length and for purity against total thickness. These curves show the relation between the purity of the transmitted energy and T_M and

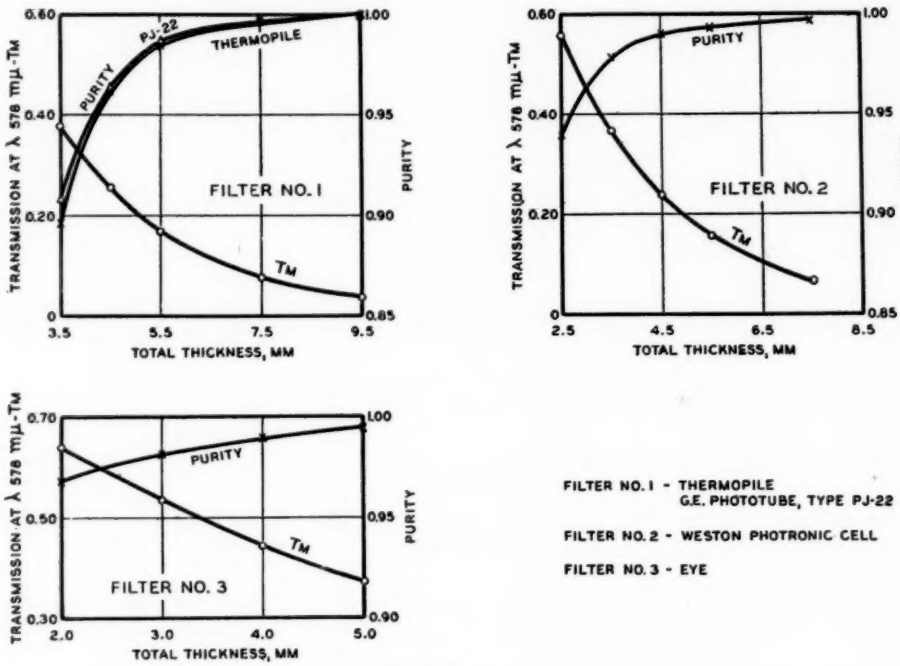

FIG. 4. Monochromats designed for wave-length 578 mμ.

from them an appropriate selection of the combination filter thickness can be made.

The wave-length 365 mμ monochromat represents a simple method of design for a two-component filter and in Fig. 2 are shown curves for the purity of the transmitted energy and for T_M for two monochromats designed for this wave-length. For any filter T_M is independent of the receiver but the purity is not. Two separate plots of filter No. 1 are given for clarity, one showing the purity curve for the thermopile, the other showing curves for two photo-cells. These monochromats have two components; component *I*, which supplies the cut-off at the lower wave-lengths, is Jena BG-10 and component *II*, which absorbs the unwanted energy at the long wave-lengths, is Jena UG-2. The most efficient thickness ratio for these glasses for use with either a thermopile, type PJ-22 photo-tube or photronic cell is 0.4. From the curves one can select the required total filter thickness. If a high purity factor, for example 0.99, is desired the available T_M and the total thickness are quickly determined. For the thermopile these will be about 0.36 and 4.0 mm, respectively. If using a photo-cell, as the type PJ-22 or the photronic cell, T_M will be greater, about 0.41, and the total thickness will be in the neighborhood of 3.5 mm. The second monochromat for 365 mμ was designed for use with the type FJ-76 photo-tube. The components are the same, but the proportions of each are different, R is equal to 0.75. At the same selected purity, 0.99, T_M for this filter is 0.325 and the total thickness is 3.5 mm. For this receiver somewhat less UG-2 is needed and half again as much BG-10.

The most efficient filter components are those having a steep slope at the cut-off adjacent to the wave-length of design and a high degree of opacity in the adjoining regions. These conditions are found only in a few types, particularly in those absorbing strongly at the shorter wave-lengths. Glasses which absorb the longer wave-lengths generally exhibit a gradual diminution in transmission and are, therefore, not very efficient. Another feature which is common to most of these glasses is an increasing transmis-

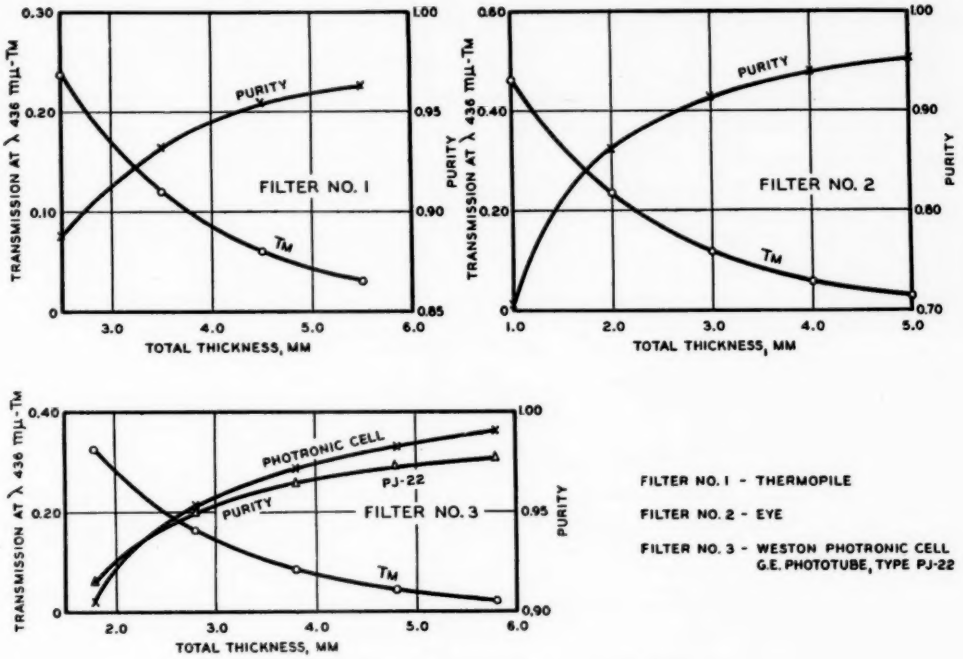

FIG. 5. Monochromats designed for wave-length 436 mμ.

sivity to radiation in the visible red and infra-red regions. To achieve an effective cut-off with filters of this type a relatively large thickness would have to be used. In order to compensate for the transmission of the second component at the longer wave-lengths, a third component filter having a high transmission at the wave-length of design and strong absorption for the undesirable radiation, was added to the combination. In this way a greater degree of purity with a correspondingly larger T_M was obtained than would have been possible with a two-component monochromat.

Monochromats for 405 mμ, shown in Fig. 3, illustrate this point. Component *I* is Jena GG-13, component *II* is Corning No. 597 red purple ultra and component *III* is Jena BG-12.[5] Both the GG-13 and the No. 597 red purple ultra have relatively sharp cut-offs at either side of wave-length 405 mμ, respectively. The latter filter, however, also has a high transmission in the red. The type FJ-76 photo-tube is insensitive to radiation above 480 mμ and for working with this cell a two-component monochromat was designed having a thickness ratio of 1.4. The thermopile, the photronic cell and the type PJ-22 photo-tube are all sensitive in varying degrees to radiation at the longer wave-lengths. For these detectors it was necessary to add the third component, BG-12, in a thickness of 2.0 mm. In designing these three-component filters the most efficient thickness for component *III* was first found and the other two components were designed in the usual way assuming as an energy distribution that transmitted through component *III*.

The components used in the design of monochromats for wave-length 578 mμ are Jena OG-2, which has a very sharp cut-off at wave-lengths 560–570 mμ and a very high transmission at wave-length 578 mμ, Jena VG-3, which has

[5] Spectral transmissions for all Jena glasses were taken from measurements supplied by Schott & Gen. The glasses used are current melts. Transmissions for Corning glasses were taken from data published in the Cleveland Chemical Rubber Publishing Co.'s *Handbook of Chemistry and Physics*, 21st edition. All results given are for the purpose of illustrating the methods of design described and showing their possibilities. These designs not only illustrate the methods but are sufficiently accurate for most purposes without recalculation.

pronounced absorption bands, and Jena BG-18, which absorbs strongly in the infra-red. An examination of the data in Table I will reveal that the intensity of the type H-3 mercury lamp is high at wave-length 578 mμ and relatively low at the longer wave-lengths. The effect of the intensities at the long wave-lengths is reduced further by the decreasing sensitivity at these wave-lengths of all of the detectors except the thermopile, which of course is nonselective.

Filter OG-2 has but one function in the combination, that of eliminating the undesired energy at wave-lengths shorter than 578 mμ, and by its character performs this most efficiently at one thickness. This thickness was readily determined. It was found that the required proportions of component filters VG-3 and BG-20 vary with the receiver used. To secure the most efficient design of monochromatic filters for wave-length 578 mμ it was, therefore, necessary to employ a different method. This differs from the methods described in that the short wave-length cut-off component is regarded as the auxiliary filter,

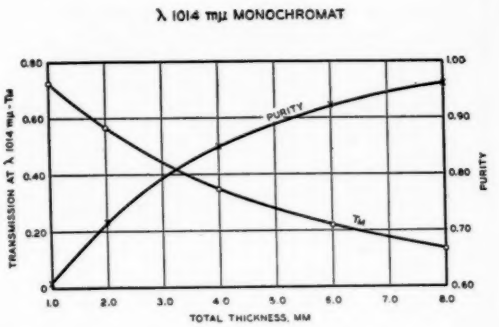

FIG. 7. Monochromat designed for wave-length 1014 mμ.

and the best thickness ratio must be determined for the remaining two components whose functions are to supply the long wave-length cut-off and to absorb the unwanted infra-red radiation. Curves for these monochromats are given in Fig. 4.

The design of the monochromat for visual work at 578 mμ presents another possibility. The thickness of component *I*, OG-2, is a con-

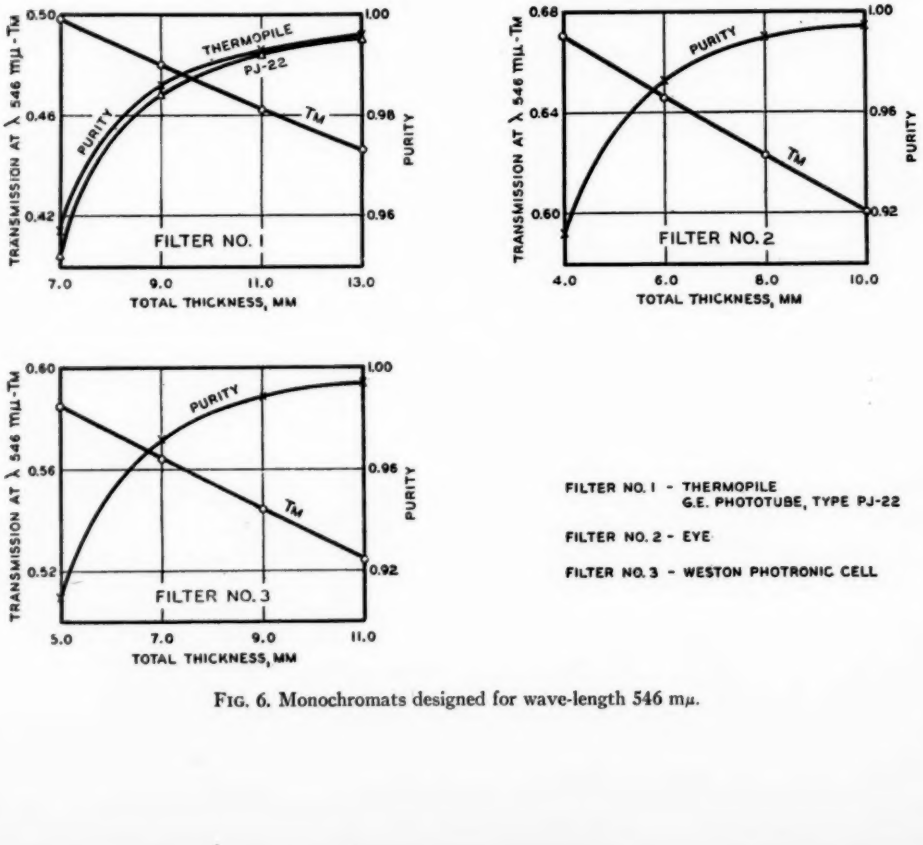

FIG. 6. Monochromats designed for wave-length 546 mμ.

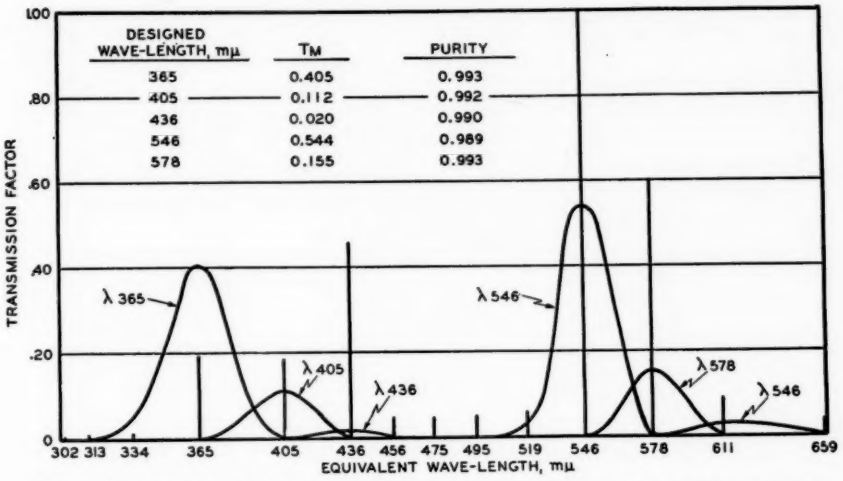

FIG. 8. Spectral transmissions of monochromats designed for use with a Weston photronic cell. The table insert gives the transmissions at the wave-lengths of design for a selected purity factor of approximately 0.99. These curves illustrate the high degree of monochromatism which can be obtained.

stant for any receiver. The thickness of the second component, VG-3, can be varied, and purities computed and curves for T_M and purity for the combination plotted for a range of practical thicknesses. The results do not differ greatly from those obtained when applying the method first described for designing two-component monochromats.

Monochromats for wave-length 436 mμ are shown in Fig. 5. Components *I* and *II* are, respectively, Corning Noviol A No. 038[6] and Jena BG-2. The sensitivity of the eye is relatively low to energy at wave-lengths above 700 mμ and the transmission of BG-2 is very small in this region. Consequently, no auxiliary filter is required for visual work at 436 mμ. For the other receptors a third component, Jena BG-7, was added to absorb the infra-red radiation transmitted by BG-2. For the thermopile a thickness of 1.5 mm of filter BG-7 was used. For the type PJ-22 photo-tube and the photronic cell BG-7 was added in a thickness of 0.8 mm. Less would probably be satisfactory but not very practical to make.

Three monochromats representing three different combinations of the components were designed for wave-length 546 mμ. Component *I* is Jena OG-1, component *II* is Jena BG-20 and the auxiliary filter, component *III*, is Jena BG-18. Filter OG-1 has a very steep cut-off at the short wave-length end at about 520 mμ. BG-20 has a sharp cut-off adjacent to 546 mμ on the long wave-length side. The proportions of these two components for all the monochromats are the same, *R* is equal to 0.25. Once this was observed the problem resolved itself into one of finding the most efficient minimum thickness for the auxiliary component BG-18. This was found to vary from 2.0 mm to 4.0 mm, depending upon the receptor. As seen from the curves shown in Fig. 6 the purities of these monochromats and the corresponding transmissions at wave-length 546 mμ are relatively high.

The monochromat for wave-length 1014 mμ is a two-component filter. The components are Jena RG-9 and Jena BG-3; *R* is equal to 1.0. Fig. 7 shows the curves for this monochromat.

The spectral transmission curves of the monochromats designed for use with the Weston photronic cell are shown in Fig. 8. A table is included in this graph which gives the transmissions at the wave-lengths of design for the various monochromats for a purity factor of approximately 0.99. The figure illustrates the high degree of monochromatism which can be

[6] Published data in the *Handbook* were used for the Noviol A. These data were understood to be for a sample having a thickness of 2 mm and were computed accordingly.

TABLE II. *Monochromats for type* H-3 *mercury lamp.*

DESIGNED WAVE-LENGTH, mμ	FILTER NO.	RECEIVER	FILTER COMPONENTS		THICKNESS RATIO, *R* (*I*/*II*)	AUXILIARY FILTERS
			I	*II*		
365	1	Thermopile	Jena BG-10, Melt No. 3624	Jena UG-2, Melt No. 3017	0.4	Water cell, 1 cm
		Weston Photronic Cell	"	"	"	—
		G. E. Photo-tube, type PJ-22	"	"	"	—
	2	G. E. Photo-tube, type FJ-76	"	"	0.75	—
405	1	Thermopile	Jena GG-13, Melt No. 24650	Corning No. 597 Red Purple Ultra	1.75	Water cell, 1 cm + Jena BG-12, Melt No. 22870, 2.0 mm
	2	G. E. Photo-tube, type FJ-76	"	"	1.4	—
	3	Weston Photronic Cell	"	"	"	Jena BG-12, Melt No. 22870, 2.0 mm
		G. E. Photo-tube, type PJ-22	"	"	"	"
436	1	Thermopile	Corning No. 038 Noviol A	Jena BG-2, Melt No. 3835	0.5	Water cell, 1 cm + Jena BG-7, Melt No. 21426, 1.5 mm
	2	Eye	"	"	"	—
	3	Weston Photronic Cell	"	"	"	Jena BG-7, Melt No. 21426, 0.8 mm
		G. E. Photo-tube, type PJ-22	"	"	"	"
546	1	Thermopile	Jena OG-1, Melt No. 24450	Jena BG-20, Melt No. 25444	0.25	Water cell, 1 cm + Jena BG-18, Melt No. S1154, 4.0 mm
		G. E. Photo-tube, type PJ-22	"	"	"	Jena BG-18, Melt No. S1154, 4.0 mm
	2	Eye	"	"	"	Jena BG-18, Melt No. S1154, 2.0 mm
	3	Weston Photronic Cell	"	"	"	Jena BG-18, Melt No. S1154, 3.0 mm
578	1	Thermopile	Jena VG-3, Melt No. 24599	Jena BG-18, Melt No. S1154	0.5	Water cell, 1 cm + Jena OG-2, Melt No. 21707, 1.5 mm
		G. E. Photo-tube, type PJ-22	"	"	"	Jena OG-2, Melt No. 21707, 1.5 mm
	2	Weston Photronic Cell	"	"	2.0	"
	3	Eye	Jena OG-2, Melt No. 21707	Jena VG-3, Melt No. 24599	2.0	—
1014	1	Thermopile	Jena RG-9, Melt No. 3947	Jena BG-3, Melt No. 17436	1.0	Water cell, 1 cm

achieved. Although the values were computed to three significant figures such accuracy is not assumed or warranted by the data used.

A summary of the data for the designed monochromats is given in Table II. This table gives the component filters, their thickness ratio, *R*, and the auxiliary filters at each designed wave-length for the five receptors considered. By the use of this table and the curves in the figures the appropriate component glasses can be selected and their thicknesses determined in accordance with most requirements.

A Recording Photoelectric Spectroradiometer

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(Received December 14, 1937)

A recording spectroradiometer has been constructed for use in absorption spectra measurements in the visible and near infra-red regions. A Hilger quartz monochromator and an emission type photoelectric cell are used. The photoelectric cell current is amplified by a d.c. amplifier and the output of the amplifier actuates a Leeds and Northrup Speedomax recorder which produces an inked record.

A RECORDING microphotometer has recently been described by the authors[1] in which a Leeds and Northrup Speedomax recorder is used as the recording element. This recorder has proved so satisfactory in the microphotometer that it seemed desirable to attempt its use as a direct recorder of absorption spectra.

The spectrometer used is a Hilger quartz monochromator and the light sensitive element is a Westinghouse type SK61 photoelectric cell. With this combination it is possible to work over the spectral range from 900 mμ to 370 mμ. The amplifier circuit shown in Fig. 1, consists of a first stage using the FP-54 electrometer tubes, which have a low grid current and can therefore be used with a large resistance in the input circuit, and a second stage using 112-A tubes, which have a high transconductance. In both stages, the ordinary balanced tube circuit is used. The first stage is so arranged that all of the potentials for the elements in the tubes are taken from a single 10 volt battery.[2] In the second stage it is necessary to use separate plate batteries for the two tubes, due to the fact that the recorder will not operate with its input terminals at an appreciable potential above or below ground potential. By shielding each vacuum tube and the photoelectric cell with iron, shielding the grid circuits from the plate circuits, and connecting the one microfarad condenser C across the input circuit of the second stage, the amplifier and recorder were made stable and free from response to outside disturbances when a standard Speedomax recorder was used.[3]

In making a record of an absorption spectrum, the wave-length drum and the focusing rack and pinion on the monochromator are driven by a synchronous motor which operates from the same a.c. line as the synchronous motor which drives the paper in the recorder. The light source is a 6-volt, 108-watt, spiral filament lamp and the absorption cell is mounted between the exit slit of the monochromator and the photoelectric cell. It requires 15 minutes for the motor to drive the wave-length drum from 900 mμ to 365 mμ and, since the paper speed for the recorder is 5 cm per minute, the record of the spectrum is 75 cm long.

In order to obtain the percent transmission of a solution as a function of the wave-length, a record is first made with the absorption cell

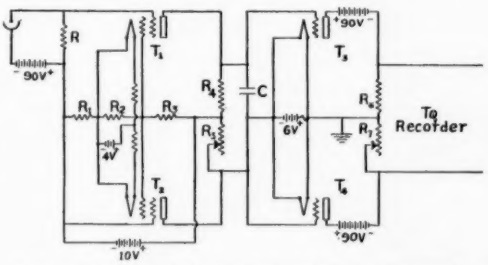

FIG. 1. Diagram of the photoelectric cell circuit and two stage d.c. amplifier. R=about 10^9 ohms, R_1=3000 ohms, R_2=5000 ohms, R_3=2000 ohms, R_4=10,000 ohms, R_5=10,000 ohm rheostat, R_6=15 ohms, R_7=15 ohm rheostat, T_1 and T_2 are FP-54 tubes, T_3 and T_4 are 112A tubes, C is a 1 microfarad condenser.

[1] H. V. Knorr and V. M. Albers, Rev. Sci. Inst. **8**, 183–184 (1937).

[2] This is essentially the circuit recommended by the General Electric Company for use with the FP-54 tubes.

[3] The Leeds and Northrup Company have now supplied us with a Speedomax recorder with a special filter in the input circuit which makes the condenser C unnecessary and improves the operation of the recorder with any d.c. amplifier.

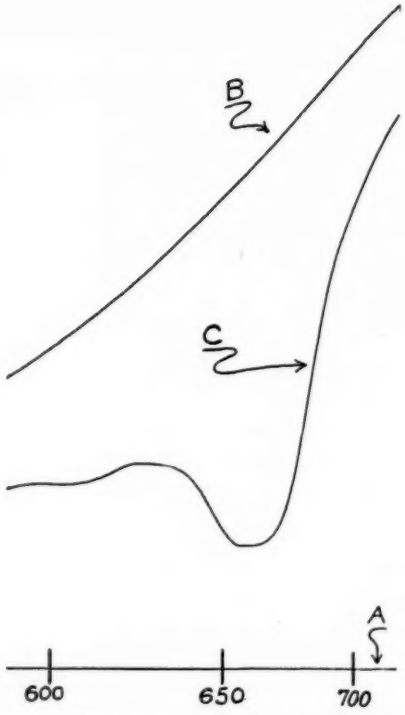

FIG. 2. A portion of a record of the absorption spectrum of chlorophyll.

filled with the solvent in the light path. A second record is then made with the cell filled with the solution. It is then possible to determine the percent transmission at each wave-length by dividing the deflection produced when the cell was filled with the solution by the deflection when the cell was filled with the pure solvent. The photoelectric cell current has its maximum value at 800 mμ and it decreases continuously as shorter wave-lengths are approached. Because of the large variation in the photoelectric cell current throughout the spectrum, it was found desirable to divide the spectrum into six different regions. Each of these regions is recorded with a diaphragm in the light path which will allow the recorder to give full scale deflection at the part of the region where the sensitivity is greatest. Each of the six regions may be studied separately or they may all be placed on the same record. In our laboratory they are all placed on one record. This is done through the use of a set of diaphragms so mounted that the successive openings may be flipped into place at the proper positions of the wave-length drum as the record is being made.

Figure 2 is a record of a portion of the spectrum of an ether solution of chlorophyll. Curve *A* was made with the slit of the monochromator covered, and represents the zero of the instrument. Curve *B* was made with the absorption cell filled with pure ether and curve *C* was made with the absorption cell filled with the chlorophyll solution. Fig. 3 is the curve showing the percent transmission of the chlorophyll solution as a function of the wave-length determined from the records in Fig. 2.

A wave-length scale with divisions ruled at

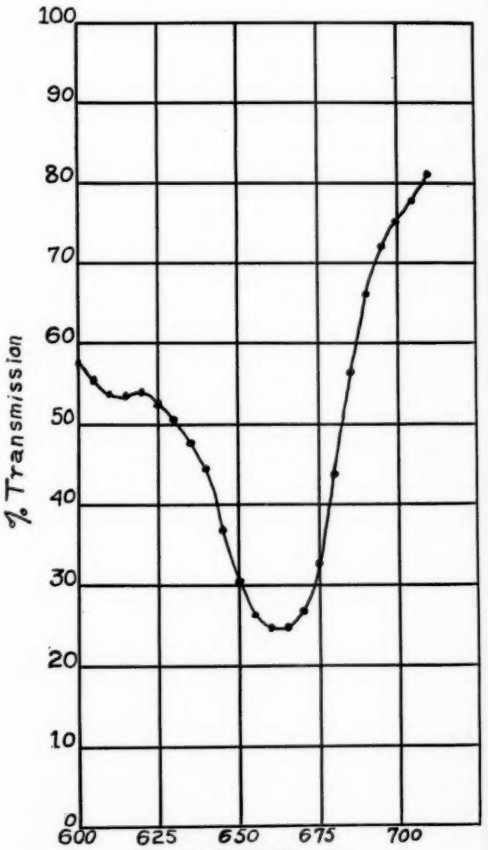

FIG. 3. Percent transmissions of a chlorophyll solution plotted as a function of wave-length computed from the records of Fig. 2.

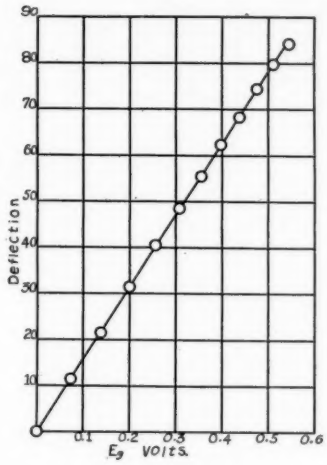

FIG. 4. Deflections of the recorder pen plotted as a function of potentials applied to the grid of the tube T_1 over the range of deflections used in making records.

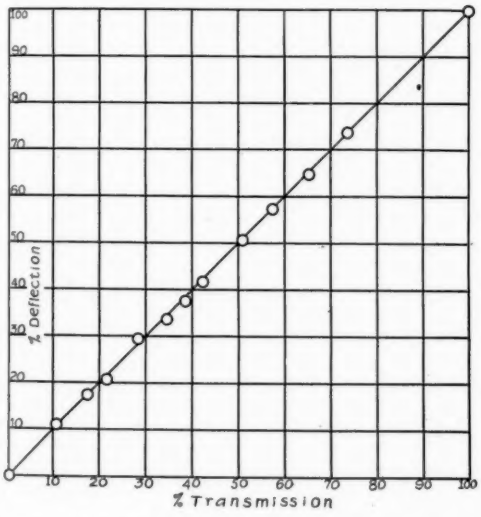

FIG. 5. Curve showing the percent deflection of the instrument as a function of the percent transmission of neutral screens placed in the light path.

25A intervals is used for analysis of the records. By illuminating the slit with the radiation from a mercury arc as the wave-length drum passes over the 365 mμ region, a record of the 365 mμ line of mercury is obtained which serves as a fiducial line from which the other wave-lengths are measured.

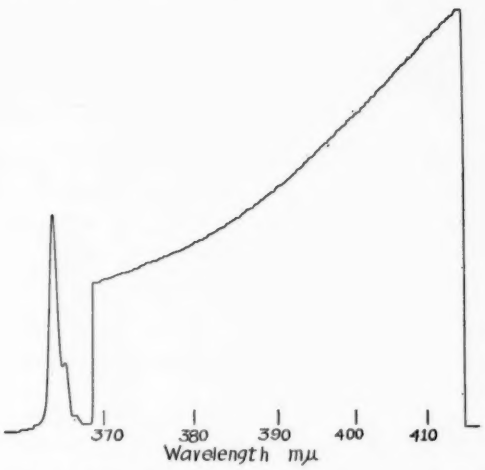

FIG. 6. A sample record of the radiation from the light source over the range from 410 mμ to 370 mμ with the mercury line at 365 mμ for wave-length determination.

Figure 4 shows deflections of the recorder pen plotted as a function of grid potentials applied to the tube T_1 over the range of deflections used in making records. Fig. 5 shows the percent deflection of the recorder pen as a function of the percent transmission of a series of wire mesh neutral screens used in the light path. The screens were calibrated by means of a General Electric PJ 22 photoelectric cell. The response of this cell has been checked against a thermocouple over the range of light intensities used and it is linear. The values of percent deflection agree with the values of percent transmission within the limits of accuracy of the determination of the values of the percent transmission for the screens used.

Figure 6 is a sample record from the violet end of the spectrum showing the mercury line at 365 mμ for wave-length determination. This record was made with no solution in the light path and therefore represents the response of the instrument to the radiation from the spiral filament lamp over this range of the spectrum.

The height of the illuminated portion of the collimator slit is 4 mm. The width of both of the slits is about 0.03 mm. The resolution obtained under these conditions is ample for measurements of absorption spectra of solutions.

A Polarizing Vertical Illuminator

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(Received February 10, 1938)

THE illumination of opaque objects for microscopic examination has been accomplished for many years by the use of a thin glass plate inclined 45° to the axis of the microscope (Fig. 1). Light from a suitable source is reflected by the two surfaces of the glass plate through the objective onto the specimen and, after reflection at the specimen, is passed back through the objective and the glass plate to an eyepiece where the specimen may be seen. There are three objections to this method of illumination: First, the large loss of light at the glass plate, only about 5 percent of the incoming light being reflected from the first surface and about 5 percent from the second surface, making a total of about 10 percent of the initial radiation being directed toward the objective. On the return of light from the specimen through the glass plate only about 90 percent is transmitted or, of the light directed on the glass plate, 9 percent of it might reach the image plane, if the objective had 100 percent transmission and if the specimen was 100 percent reflecting. Second, the glass plate is in a beam of converging light and consequently because it is a plano parallel inclined plate, will introduce a small amount of astigmatism in the image. Third, the examination of metal specimens in plane polarized light is complicated by the fact that a polarizing prism must be placed between the light source and the plane glass reflector. An analyzer must be placed either on top of the eyepiece which limits the field of view or between the objective and eyepiece where it is mechanically difficult to introduce it and where, to secure images free from astigmatism, it must be equipped with astigmatizing lenses, or the objectives must be corrected for infinity and a telescope objective placed on the eyepiece side of the prism to focus the image in the focal plane of the eyepieces. The most serious objection to polarized light applied to the plane glass vertical illuminator is that the plane glass illuminator introduces elliptical polarization and only a narrow region of the field of view will be perfectly extinguished when the two prisms have their planes of vibration 90° apart.

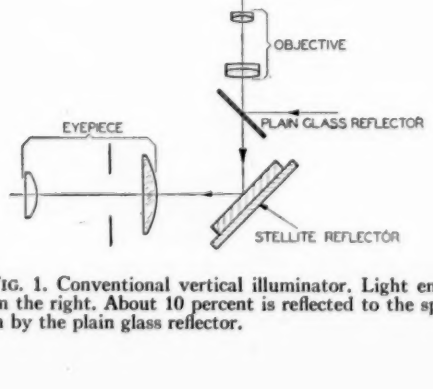

FIG. 1. Conventional vertical illuminator. Light enters from the right. About 10 percent is reflected to the specimen by the plain glass reflector.

I have made a vertical illuminator which is more efficient than the plane glass type, has no inclined plano parallel plate in the path of light from the specimen to the image, and allows the specimen alone to reflect light to the image plane. This illuminator is made of calcite (Fig. 2). It consists of two parts, *A* and *B*. These parts are made from one piece of calcite with the optic axis parallel to the end of the prism and in the plane of the cemented surface. The two parts are cemented together with a cement having approximately the same index of refraction as the extraordinary ray. Light from the source travels along the axis of the microscope objective and, as in a Nicol prism, is divided on entering the calcite into two rays, one the ordinary and the other the extraordinary. The ordinary is reflected and absorbed by black paint on the side of the prism while the extraordinary passes on through the prism, through the objective and to the specimen. The plane of vibration of this light which reaches the specimen lies in the plane of the drawing. The light illuminating the specimen is plane polarized and a perfect reflecting specimen would reflect the plane polarized light unaltered and it would pass back through the prism along its original path.

Most metal specimens are not entirely specular, especially after they have been etched to bring out their particular grains and the crystalline structure of them. Where the etchant has dissolved away the material, there is left an uneven surface composed of numerous grooves, the sides of which are inclined to the general plane of the specimen. One can prove by Fresnel's equations that the sides of these grooves will reflect plane polarized light and when illuminated by plane polarized light will in most cases rotate the plane of polarization of the light incident on them. Therefore, from the usual metal specimen, there will be certain details which will reflect the plane polarized light but after reflection its vibration plane will not lie in the plane of the drawing. This light reflected by the specimen will be partially reflected at the cemented layer in the calcite prism to the silvered surface where it is reflected across the prism and to an eyepiece. All regions in which the light is reflected specularly will appear dark since the vibration plane of this reflected light is still in the plane of the drawing and none of it will be reflected at the cemented layer. The prism thus acts as a polarizer for light incident on the specimen and as an analyzer for light reflected from the specimen. It should be pointed out here that the light reflected at the cement calcite boundary follows the laws of refraction of the ordinary ray since this is the only ray which can be reflected. The index of calcite for the ordinary ray is 1.6585 and the index of the cement is 1.4864. Thus there can be no astigmatism along the path the light travels from the objective to the eyepiece.

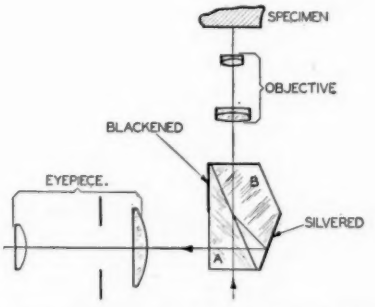

FIG. 2. Polarizing vertical illuminator. Light enters the calcite prism from below, is divided into the ordinary and extraordinary rays. The ordinary ray is reflected at the dividing surface between *A* and *B* and the extraordinary ray is transmitted to the specimen. A perfect reflecting specimen would reflect the plane polarized light through the prism back to the light source. Depolarization at the specimen will reflect light which may be vibrating in a plane perpendicular to the plane of the figure. This light will be reflected at the dividing surface between *A* and *B* and again at the silvered surface to the eyepiece where the specimen will become visible. This examination is equivalent to the examination of a transparent specimen between crossed Nicols.

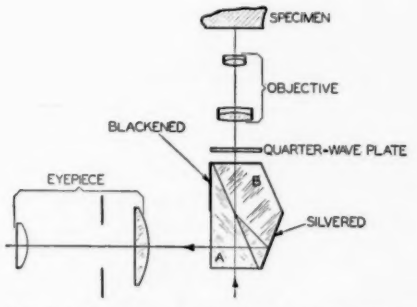

FIG. 3. Polarizing vertical illuminator with quarter-wave plate. Introduction of the quarter-wave plate is equivalent to introducing a half-wave plate between a polarizer and analyzer since the light passes through this plate twice. This gives bright field illumination.

This prism illuminator can also be used to view metal surfaces in bright field illumination, i.e., so that specular regions appear bright and grooves such as result from etchants appear black. Bright field illumination is secured by placing between the prism and the objective a quarter-wave plate with its slow ray direction making an angle of 45° with the vibration plane of the prism (Fig. 3). The plane polarized light after emergence from the prism passes through the quarter wave plate and becomes circularly polarized. It is reflected by the specimen and again passes through the quarter-wave plate. It has thus had a 90° change of phase and is now vibrating perpendicularly to the plane of the drawing. This light is fully reflected at the cement calcite layer to the silvered surface and to the eyepiece. The specimen appears in this kind of illumination exactly as it does when illuminated with the customary plane glass illuminator. It is to be observed, however, that of the initial intensity of light 50 percent is transmitted through the prism. There is about a 20 percent loss by two transmissions through the quarter-wave plate which makes the intensity of the light about 40 percent of its initial quan-

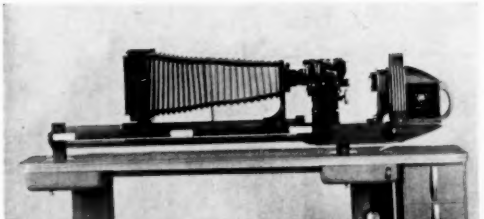

FIG. 4. Research metallographic equipment embodying polarizing vertical illuminator.

tity when it enters the prism after having been reflected by the specimen. All of this light will be reflected at the two reflecting surfaces to the eyepiece. Thus, this prism is about 4½ times as efficient as the plane glass reflector.

The three objections to the plane glass illuminator, as mentioned above, are not present in this polarizing vertical illuminator. First, the efficiency of the latter is about 4½ times that of the former; second, there is no astigmatism present because the image from the objective enters the prism and leaves it at normal incidence, and is reflected twice as plano surfaces; third, the illuminating prism is both a polarizer and an analyzer.

Figure 4 shows a microscope which has recently been built incorporating this new prism. The light source of this microscope is a new lamp developed at Nela Park. It has all of the electrical characteristics of the S_1 sun lamp but one of the electrodes has been made with a solid rod of tungsten which glows at a very high temperature and furnishes a small homogeneous source of light of very high intensity. This microscope is equipped with a binocular observation eyepiece, adjustable for inclination, a ball bearing revolving stage with built-in mechanical stage, and has both coarse and fine adjustment, remote control accessible any place along the camera bed. The polarizing vertical illuminator is contained in a turret which also contains a cube of glass for dark field illumination. Without changing focus, one may change style of illumination from bright field to either polarized light or to dark field.

A Study of Polarized Vertical Illumination

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(Received February 10, 1938)

THE desire to illuminate opaque specimens with incident light has come from the minerologist and the metallographer. Dr. F. E. Wright in his paper on the "Examination of Ores and Metals in Polarized Light" had discussed methods using a prism illuminator and a Nicol prism. Such an illuminator covers up one-half of the microscope objective and illuminates the object with an oblique pencil of light. The uncovered half of the objective is used to form the image of the illuminated object. This illuminator has been used because with the plane glass illuminator the illumination is already weak and is further reduced by the introduction of a Nicol prism polarizer in the illumination beam. There are several limitations to the effectiveness of polarization whichever type of illuminator is used.

When the prism illuminator is used with high power objectives the unsymmetrical pencil of light coming from the objective leads to a poorly defined image quite unsuitable for most metallographic purposes. Whichever illuminator is used, the polarizer can only be placed with its vibration plane perpendicular to the plane of incidence or parallel to it. This condition is made with the assumption that plane polarized light is to fall on the specimen so that when the latter is examined with the microscope equipped with an analyzer, extinction will exist in the center of the field. This, furthermore, assumes that the surface under examination specularly reflects the plane polarized light without change of phase. If, as is usually required, the plane polarized light is rotated to determine the optical character of the crystal grains of the specimen, the light falling on the specimen will be elliptically polarized and it will be impossible to locate the analyzer in any meridian to give extinction. Furthermore, even when the vibration plane of the polarizer is perpendicular or parallel to the plane of incidence the entire field of the microscope will not be black when the analyzer is set for maximum extinction at the center of the field. The field will have across its center and parallel to a plane containing the axes of the microscope and the illuminating beam a band about 1° wide (real field of the microscope) in which extinction exists. The total real field of the microscope is about 6°.

The purpose of this paper is to explain the reasons for the second and third limitations to placing a polarizer between the light source and either the plane glass vertical illuminator or the various mirror illuminators. In order to understand the reason for the balance of the field departing from extinction it is necessary to refer to the Fresnel equations which give the amplitudes of vibration when a beam of unhomogeneous light falls at any angle (i) on the boundary between to media (Fig. 1). In these equations E_s and E_p represent the electric forces perpendicular to and parallel to the plane of incidence, respectively; R_s and R_p are the amplitudes of the reflected vibrations perpendicular to and parallel to the plane of incidence. i and i' are the angles of incidence and refraction at the surface.

$$R_s = -E_s \frac{\sin(i-i')}{\sin(i+i')}, \quad R_p = E_p \frac{\tan(i-i')}{\tan(i+i')}.$$

Suppose the reflector is a plane glass plate and by considering only the top of the plate, i.e. the air glass surface where the glass has an index of refraction of $n=1.5$ and let $i=45°$. Then if the light is unpolarized and $E_s=E_p=1$.

$$R_s=-0.3032, \qquad R_p=0.0919.$$

This shows that the upper surface of the plane glass reflector is a partial polarizer. Now, if the original light reaching the reflector is polarized in either the plane of incidence or in a plane perpendicular to the plane of incidence, either E_p or E_s is zero and the light reflected by the reflector is also polarized in the same plane as before reflection. However, if the plane of polarization of the polarized light is rotated through an angle, say 45°, which can be accomplished by rotating the Nicol prism polarizer, what will the state of polarization of the reflected light be? The light which reaches the reflector can be

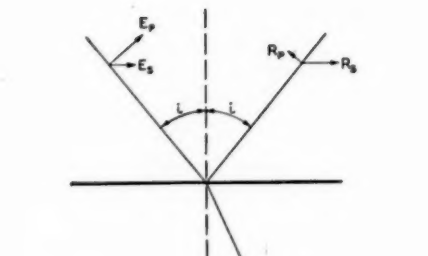

FIG. 1. Ordinary light coming from the left is incident on the boundary between two media. The reflected light and transmitted light are partially polarized.

resolved into the two components parallel with the plane of incidence and perpendicular to it (Fig. 2), then $E_s = E_p$ and for the same conditions as above if the original amplitude is 1

$$R_s = -\tfrac{1}{2}\sqrt{2} \times 0.3032, \quad R_p = \tfrac{1}{2}\sqrt{2} \times 0.0919.$$

The light is still plane polarized, but before reflection had amplitude of $\frac{1}{2}\sqrt{2}$ in the two principal meridians and after reflection is oriented at $\tan^{-1} 0.3032/0.0919$ or 73° 8′ away from the plane of incidence. If we denote E_p/E_s by $\tan x$ and R_p/R_s by $\tan x'$ we can divide the Fresnel equations for R_p by R_s and get

$$\tan x' = -\tan x \frac{\cos(i+i')}{\cos(i-i')},$$

x will denote the angle the plane of polarization makes with the plane of incidence before reflection and x' the angle after reflection.

If $x = 5°$, $x' = 16°\ 6'$,
$x = 1°$, $x' = 3°\ 18'$,

and we see that for a small rotation of the plane of polarization in the incident beam, the reflected plane of polarization is rotated considerably.

Now to get back to the illumination of the microscope. It is easily understood that the beam of light illuminating the field of the microscope cannot be parallel and illuminate a field of real size, but must converge towards the objective and diverge to the specimen. There are, therefore, illuminating pencils of light which must be reflected by the reflecting prism in which the plane of incidence is not parallel with the plane of incidence of the axial pencil of the illuminating beam. For instance, in the plane containing the axis of the illuminating pencil but perpendicular to the axis of the microscope, the rays from the edge of the light source form with their normals at the reflector planes not parallel with the plane of incidence of the axial ray. The vibration planes for these reflected rays will therefore not be parallel with that through the center. However, in the plane containing the axis of the microscope and the axis of the illuminating beam all rays will lie in the same plane of incidence. Thus it can be seen that the field will be illuminated by plane polarized light but only in one region will the vibration direction lie orthogonal to the direction of the illuminating beam and it will be possible to extinguish objects within the field only in a narrow band at one time. The analyzer can be rotated and the region of extinction will move or brush across the field as it is rotated.

If the polarizer and analyzer are rotated for the purpose of determining whether or not grains in a specimen are producing phase changes, the analyzer cannot be rotated simultaneously with the polarizer because as has been shown the

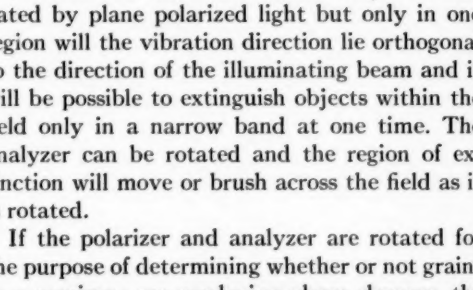

FIG. 2. "E" represents the amplitude and direction of vibration before reflection, "E_P" and "E_S," electric forces parallel to and perpendicular to the plane of incidence respectively. "R" is the amplitude and direction of vibration after reflection.

reflected plane of polarization is not rotated through the same angle as the incident plane of polarization.

Dr. Max Berek of Wetzlar, Germany, has invented two devices for increasing the field of polarization of reflecting vertical illuminators. One of these is a wedge mirror and the other, described in *Zeitschrift für Instrumentenkunde* 1936, is a multiple reflecting prism. He gives no values for the accomplishments of the wedge mirror but for the multiple reflecting prism he claims that if the plane of polarization of the incident beam is rotated 1°, from the plane of incidence, the plane of polarization of the reflected beam will be rotated at a maximum 0.08° and if the plane of polarization of the incident beam is rotated 45° from the plane of incidence the plane of polarization of the reflected beam will be rotated a maximum of 0.05°. The values are given for a prism of ^{n}D 1.740. If the prism is made of glass whose ^{n}D is 1.516, these rotations become 6.05° and 7.63°, respectively.

These devices both utilize only the half-aperture of the objective. Furthermore, since they only deal with the illumination of the object no reflecting surface should come between the object and the analyzer or the same depolarizing effects will occur in the microscope path.

In 1935, the writer described a polarizing vertical illuminator before this society. This illuminator[1] consisted of a special construction of a calcite prism in which the ordinary ray was reflected and made use of for transmitting the path of light from an objective to an eyepiece and the extraordinary ray was used for transmitting plane polarized light to the specimen. Since this time, the prism has been made in a different manner, by making use of two glass wedges and a layer of calcite so that the ordinary ray may be transmitted and the extraordinary ray reflected for illumination purposes. The prism is shown in Fig. 3. Unpolarized light from a light source *LS* passes through the lens and enters the glass prism passing through the thin calcite plate and is reflected at the silvered

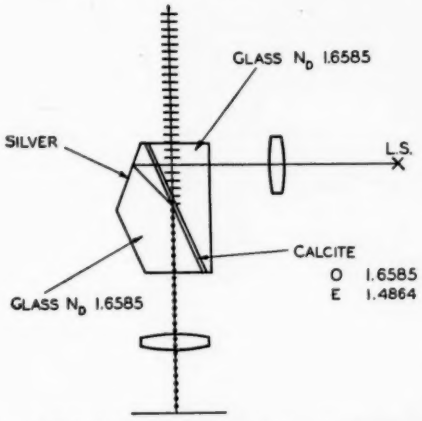

FIG. 3. Polarizing vertical illuminator. Unhomogeneous light is polarized at the glass calcite boundary. The extraordinary ray is reflected and the ordinary ray is transmitted.

surface to the calcite plate where a part of it vibrating in a plane perpendicular to the plane of incidence is reflected to the objective. The other part is transmitted and absorbed at the lower part of the entering face which is ground and blackened. The light reflected by the specimen passes back through the objective and since the transmitted ray at the calcite layer must vibrate in the plane of incidence after transmission, no light will be transmitted unless it has been depolarized by the specimen. There will be complete polarization in all parts of the field since the reflecting surface of the calcite is the polarizer and it is inclined at an angle sufficient to include the angular field of the microscope. Since the polarizer and analyzer are one piece a rotation of the lamp and prism provide simultaneous rotation of polarizer and analyzer. Or, if the microscope is provided with a rotating stage, the specimen can be rotated in place of rotating the prism and lamp. The rotation of the specimen will be the equivalent of rotating the plane of polarization of the illuminating beam.

With this prism the full aperture of the objective is used for both the illuminating pencil of light and the image forming beam. It therefore has none of the limitations described in the first part of this paper.

[1] See Fig. 2 of preceding article.

Flare in Schmidt Cameras

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The flare pattern obtained when a heavy over-exposure is made with a Schmidt camera on a bright object well off-axis consists of a secondary image, halos about the primary and secondary images and an annulus with these images at extremities of a diameter. The bearing of this flare pattern on the design of a Schmidt camera is discussed and it is shown how one may test a Schmidt camera by the use of its flare pattern.

A RELATIVELY simple flare pattern is obtained with a Schmidt camera[1] when a heavy over-exposure is made on a bright object, such as the full moon, placed well off-axis. In Fig. 1 is shown a sketch of such a flare pattern obtained with the Ladd Observatory Schmidt camera of four inches aperture and of focal ratio $f:1$. In this paper will be presented a description of the flare pattern, an explanation of the origin of each part of it and an indication of the manner in which the pattern can be used in testing the camera producing it. It will be assumed throughout that the correcting plate has been made with its front surface plane and with the rear surface concave at the edge, convex at the center in the fashion required to minimize the chromatic aberration of the system. The advantages of this design will be pointed out later.

Of the light rays coming from a distant bright object and entering the correcting plate, most will pass through the optical system in the intended fashion and, arriving at the photographic film, will form an image of the bright object. This image we shall speak of as the primary image to distinguish it from another well-focused image.

Some of the rays, after entering the correcting plate, will be reflected at the rear surface and of these, some will be reflected again at the front surface and will return through the optical system to the photographic film. If the plane tangent to the rear surface at the point of reflection for a given ray makes an angle, α, with the front surface (which has been assumed plane, and is now assumed perpendicular to the optical axis of the system), the ray after reflection will make an angle, 2α, with the direction it possessed after first entering the correcting plate. If α_0 is the maximum value of α for all points of the rear surface of the correcting plate, then around the primary image will be an annular region, $2\alpha_0$ in width, affected by these rays, hereafter called the halo about the primary image. Here the measurements of the width of this annular region are to be interpreted in terms of the scale of the photograph, expressed in angular measure, as calculated from the focal length of the system. An examination of the form of the Schmidt correcting plate of good design will show that one should not expect this halo to be of uniform density but smoothly to decrease and then to increase in density as one moves toward the edge of the halo.

Because of the brilliance of the primary image in Schmidt cameras of large relative aperture, a considerable amount of light will be reflected at the surface of the film and will return through the optical system, some being reflected at the rear surface of the correcting plate, some at the front surface and some passing completely out of the system. Consider for the moment the light reflected at the film surface and turned again into the optical system at the front surface of the correcting plate. As this light reaches the front surface of the correcting plate, the rays will be moving in a direction parallel to, but opposite in sense to the incoming rays in the same region. After reflection, these rays will still form a parallel bundle which will return again to the photographic negative, forming there a sharp image of the bright object at a point symmetrical to the primary image with respect to the optical axis

[1] Those readers not well acquainted with the Schmidt camera may be referred to the following papers: B. Strömgren, "Das Schmidtsche Spiegelteleskop," Vierteljahrschrift der Astronomischen Gesellschaft, **70**, 65–86 (1935). C. H. Smiley, "The Schmidt Camera," Popular Astronomy **44**, 415–421 (1936).

of the camera. This image we may call the secondary image.

It will be seen that the rays reflected from the photographic film at the primary image and again at the rear surface of the correcting plate will produce another annular area $2\alpha_0$ in width, this one about the secondary image. This would naturally be called the halo about the secondary image.

Next we may well consider what happens when rays already within the optical system are reflected at the surface of a cylinder whose axis coincides with the optical axis of the system. This may be either the interior or the exterior of such a cylinder. Reflections of this sort might occur at the inner surface of a cylindrical jacket of the camera, or at the outer edge of the film holder. In the former case, the use of a larger cylinder with proper baffles painted a dead black would eliminate the difficulty. In the latter case, the edge of the film holder might be beveled sufficiently to avoid the difficulty. It will be seen that resulting from the reflections at the interior surface at such a cylinder, there will be a single infinity of overlapping images which will lie on a semi-circle with center at the focal point for paraxial rays and with radius equal to the angle by which the bright object is off-axis. From reflections at the exterior surface of such a cylinder, there will result another semi-circle, similar to and completing the circle of the first. We may call this circle the cylindrical circle.

Thus we have on our film the primary image, the secondary image, the halo about each, and the cylindrical circle.

One might consider using a spherical surface of moderate curvature as the front surface of the correcting plate in order to prevent the secondary image from being sharply focused. This would be in conformity with standard practise in lens design; usually a slight background fog and the resulting loss in brilliance in negatives is to be preferred to a well-focused ghost. However since one of the great advantages of Schmidt cameras is the extraordinary speeds to which they can be built, and since this advantage is most appreciated in photographing faint objects, it seems that one might well tolerate a sharply focused ghost which will obligingly move into any desired position, in order to have a background free from flare for the faint objects under scrutiny.

It may also be noted that the same condition which provides a minimum of chromatic aberration in the Schmidt camera will also minimize the size of the halos about primary and secondary images. The designer of a Schmidt camera should remember that whereas the usual problem is to minimize the spherical aberration of a lens or a system of lenses, here the problem is to introduce a specific amount of spherical aberration of a specific sign in a single lens in the fashion least disturbing to the system of which it is a part.

Before proceeding to show how the flare pattern of a Schmidt camera can be used in testing certain of its alignments, it may be well in the interest of completeness to make a few remarks in general about the adjustment of a Schmidt. Any person who has had much experience in the construction or adjustment of a Schmidt camera will be familiar with these facts. The angle which the correcting plate makes with

FIG. 1. The flare pattern of a Schmidt camera consists of a single bright "ghost," halos about the primary image and ghost, and a circle through them.

the axis of the spherical mirror is not critical, i.e., need not be precisely 90°. This amounts to saying that a Schmidt plate will work well over a larger field than that for which it has ordinarily been used. Nor is the position of the center of the correcting plate along the axis of the spherical mirror especially critical. This explains why several of the smaller Schmidt cameras now in use are successfully focused by moving the spherical mirror back and forth along its axis while the

correcting plate and the film holder are left fixed in position. It is essential however that the center of the correcting plate should lie precisely on the axis of the spherical mirror. If it is displaced to one side or the other, a "one-way coma" will afflict the star images all over the field, i.e., the comatic tails will all point in the same direction. The reason for this will be seen at once when one realizes the primary purpose of the correcting plate is the removal of the spherical aberration which would be present if the spherical mirror were used alone.

To simplify the discussion of the use of the flare pattern in tests of a Schmidt camera, we shall assume that there is only one maladjustment or one defect at a given time, all other adjustments and conditions being perfect. The practical problem will be slightly complicated by the simultaneous presence of various maladjustments and defects of the optical parts making up the system.

If the correcting plate is not perpendicular to the axis of the spherical mirror, it will be found that the point on the photographic film midway between the primary and secondary images (the focal point for the parallel bundle of rays striking the plane front surface of the correcting plate perpendicularly) will not coincide with the center of the film. Nor will the halos be circular in shape with centers in the primary and secondary images. The cell containing the correcting plate may well be held at the proper distance from the spherical mirror by three threaded rods; on each rod, nuts on either side of a flange of the cell for adjustment purposes. A similar arrangement for the spider holding the film holder is almost indispensable. It will be advantageous if these rods are made of a material of low thermal coefficient of expansion such as invar.

If the correcting plate is too near to, or too far from the spherical mirror, there will result a small amount of ordinary spherical aberration and coma. If the correcting plate be lacking in the requisite symmetry about its center, the deficiency will be discovered by a careful examination of the halos about the primary and secondary images. These halos should be symmetrical about their centers except for effects due to silhouetting by parts of the camera, as for example, by the arms of the spider supporting the film holder. The results of such silhouetting is most noticeable when the bright object is farthest from the axis of the camera.

In conclusion, it may be pointed out that when the bright object is placed on the axis of the camera, the primary and secondary images merge into one, as do the primary and secondary halos. The cylindrical circle vanishes in this case.